

# DELAY EQUATIONS FOR EPIDEMIC MODELS: INSTABILITY DUE TO WANING IMMUNITY

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ABSTRACT. The talk will begin with introduction of "delay equation formulation" of structured population models using the Kermack and McKendrick epidemic model as an example [1, 2]. We briefly review analytical results (such as a threshold dynamics and final size relation) and then extend the Kermack and McKendrick model to describe reinfection due to waning immunity of recovered individuals. Together with stability analysis, it is suggested that recovered period is the source of destabilisation of the system. We also discuss global stability of equilibria and periodicity of a periodic solution for specific cases. Comparison of two models will illustrate importance of the immune period and its distribution in the periodic oscillation of infectious diseases. The talk is based on the collaboration studies with R. Omori and G. Röst [3, 4].

## REFERENCES

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