ON LARGE EXPONENT BEHAVIOR OF POWER CURVATURE FLOW ARISING IN IMAGE PROCESSING

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Abstract. Motivated by applications in image processing, we study asymptotic behavior for the level set equation of power curvature flow as the exponent tends to infinity. More precisely, we consider the level set equation

$$u_t - |\nabla u| \left(\operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) \right)^{\alpha} = 0 \quad \text{in } \mathbb{R}^n \times (0, \infty) \tag{1}$$

with a Lipschitz initial value u_0 in $\mathbb{R}^n$. Here $\alpha > 0$ is a given exponent. It is well known [2] that for any given $\alpha > 0$ there exists a unique viscosity solution u^α of the initial value problem associated to (1). We are particularly interested in the limit behavior of u^α as $\alpha \to \infty$, which has important applications in image denoising [1].

If u_0 satisfies

$$\begin{aligned} &\text{(Quasiconvexity)} && \{x \in \mathbb{R}^n : u_0(x) \le c\} \quad \text{is convex for any } c \in \mathbb{R} \text{ and} \\ &\text{(Coercivity)} && \inf_{|x| \ge R} u_0(x) \to \infty \quad \text{as } R \to \infty, \end{aligned}$$

the limit equation can be characterized as the following stationary obstacle problem involving 1-Laplacian:

$$\min \left\{ - \operatorname{div} \left(\frac{\nabla U}{|\nabla U|} \right) + 1,\ U - u_0 \right\} = 0 \quad \text{in } \mathbb{R}^n. \tag{2}$$

We discuss various properties of the obstacle problem and show that $u^\alpha(\cdot, t) \to V$ locally uniformly in $\mathbb{R}^n$ as $\alpha \to \infty$ for any $t > 0$, where V is the minimal supersolution of (2). We also discuss the large exponent asymptotics for non-convex initial values and applications related to a math model describing unstable sandpiles. This talk is based on joint work [3] with Professor N. Yamada at Fukuoka University.

References

[1] F. Cao. *Geometric curve evolution and image processing*, volume 1805 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2003.

[2] H. Ishii and P. Souganidis. Generalized motion of noncompact hypersurfaces with velocity having arbitrary growth on the curvature tensor. *Tohoku Math. J. (2)*, 47(2):227–250, 1995.

[3] Q. Liu and N. Yamada. An obstacle problem arising in large exponent limit of power mean curvature flow equation. *preprint*.