Direct and inverse bifurcation problems for semilinear equations

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We consider the bifurcation problem

$$\begin{aligned} -u''(t) &= \lambda\left(u(t) + g(u(t))\right), \quad x \in I := (-1, 1), \\ u(t) &> 0, \quad t \in I, \\ u(-1) &= u(1) = 0. \end{aligned}$$

Here, $\lambda > 0$ is a bifurcation parameter. The typical examples of $g(u)$ are: $g(u) = g_1(u) := \sin\sqrt{u}$, $g_2(u) := \sin u^2 (= \sin(u^2))$. Then it is well known that under the suitable conditions on $g(u)$, λ is parameterized by the maximum norm $\alpha = \|u_\lambda\|_\infty$ of the solution u_λ corresponding to λ and is written as $\lambda = \lambda(g, \alpha)$. It should be mentioned that if $g(u) = g_1(u)$, then this problem has been proposed in Cheng [2] as an example which has arbitrary many solutions near the line $\lambda = \pi^2/4$. In this talk, we show that the bifurcation diagram of $\lambda(g_1, \alpha)$ intersects the line $\lambda = \pi^2/4$ infinitely many times by establishing the precise asymptotic formulas for $\lambda(g_1, \alpha)$ as $\alpha \to \infty$. We also establish the precise asymptotic formulas for $\lambda(g_i, \alpha)$ $(i = 1, 2)$ as $\alpha \to \infty$ and $\alpha \to 0$. We also treat the other nonlinear term $g(u)$. We apply these results to the new concept of inverse bifurcation problems.

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