An existence and uniqueness result for the Navier-Stokes type equations on the Heisenberg group

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Abstract

The partial differential equations associated to the sublaplacian $\mathcal{L}$ provided by the left invariant vector fields on the Heisenberg group $\mathbb{H}^d$ has studied by some mathematicians (for example, [1], [2], [3], [4], [5] and so on). The aim of this talk is to give an existence and uniqueness of solutions for the Cauchy problem of the following Navier-Stokes type equations associated to the sublaplacian $\mathcal{L}$ provided by the left invariant vector fields on the Heisenberg group $\mathbb{H}^d$,

$$\begin{cases} \boldsymbol{u}_t + \mathcal{L}\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \tilde{\nabla}\pi = 0, \\ \mathrm{div}_R\boldsymbol{u} = 0, \\ \boldsymbol{u}(g,0) = \boldsymbol{a}(g) \end{cases}$$

for $g \in \mathbb{H}^d$ and $t > 0$, limited to Lebesgue spaces on 2-step stratified Lie groups, especially Heisenberg group. The vector of the right-invariant vector fields is denoted by $\tilde{\nabla}$ and the divergence of the right invariant vector fields is denoted by div_R. To avoid the difficulty of the non-commutative which is intrinsic in 2-step stratified Lie groups, we construct the solenoidal space by using the right invariant vector fields on the Heisenberg group. We also would like to discuss what kind of natural phenomena this system reveals.

References

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