Genealogies and ages of cultural traits: An application of the theory of duality to the research on cultural evolution

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Abstract

A finite-population, discrete-generation model of cultural evolution is described, in which multiple discrete traits are transmitted independently. In this model, each newborn may inherit a trait from multiple cultural parents. Transmission fails with a positive probability unlike in population genetics. An ancestral process simulating the cultural genealogy of a sample of individuals is derived for this model. This ancestral process, denoted by $\mathscr{M}_-$, is shown to be dual to a process $\mathscr{M}_+$ describing the change in the frequency of a trait. The age-frequency spectrum is defined as a two-dimensional array whose (i, k) element is the expected number of distinct cultural traits introduced k generations ago and now carried by i individuals in a sample of a particular size n. Numerical calculations reveal that the age-frequency spectrum and related metrics undergo a critical transition from a phase with a moderate number of young, rare traits to a phase with numerous very old, common traits when the expected number of cultural parents per individual exceeds one. It is shown that $\mathscr{M}_+$ and $\mathscr{M}_-$ converge to branching or deterministic processes, depending on the way population size tends to infinity, and these limiting processes bear some duality relationships. The critical behavior of the original processes $\mathscr{M}_+$ and $\mathscr{M}_-$ is explained in terms of a phase transition of the branching processes. Using the results of the limiting processes in combination, we derive analytical formulae that well approximate the age-frequency spectrum and also other metrics.