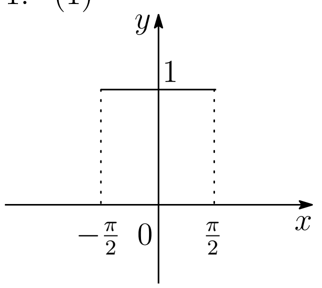
応用数学 II　No.7 解答

1.　(1)



(2)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\ dx = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin nx\ dx = \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

(3)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ dx = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\ dx = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos nx\ dx = \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

従って、$a_{2m} = 0\ (m = 1, 2, \cdots),\ a_{2m+1} = \dfrac{2(-1)^m}{(2m+1)\pi}\ (m = 0, 1, \cdots)$

2.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\ dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\ dx = \frac{1}{\pi}\left\{\left[x\frac{\sin nx}{n}\right]_{-\pi}^{\pi} - \frac{1}{n}\int_{-\pi}^{\pi}\sin nx\ dx\right\} = -\frac{1}{n\pi}\left[-\frac{\cos nx}{n}\right]_{-\pi}^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\ dx = \frac{1}{\pi}\left\{\left[-x\frac{\cos nx}{n}\right]_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos nx\ dx\right\}$$

$$= -\frac{2}{n}\cos n\pi + \frac{1}{n\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{\pi} = \frac{2}{n}(-1)^{n+1}$$

従って、$f(x) = 2\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$

3. $f(ax)$ の周期を T とすれば、$aT = p$ より $T = \frac{a}{p}$ である。従って、$\cos\frac{x}{10}$ の周期は 20π である。