応用数学 II No. 2 解答

1. $F(s) = \mathscr{L}(f)$ とする。$s > 0$ のとき、$\lim_{t\to\infty} e^{-st}\cos t = 0$, $\lim_{t\to\infty} e^{-st}\sin t = 0$.

(1)

$$\mathscr{L}(f) = \int_0^\infty e^{-st}\sin t\ dt = \left[-e^{-st}\cos t\right]_0^\infty - s\int_0^\infty e^{-st}\cos t\ dt$$

$$= 1 - s\left(\left[e^{-st}\sin t\right]_0^\infty + s\int_0^\infty e^{-st}\sin t\ dt\right) = 1 - s^2F(s)$$

従って、$F(s) = \dfrac{1}{s^2+1}$

(2)

$$\mathscr{L}(f) = \int_0^\infty e^{-st}\cos t\ dt = \left[e^{-st}\sin t\right]_0^\infty + s\int_0^\infty e^{-st}\sin t\ dt$$

$$= s\left(\left[-e^{-st}\cos t\right]_0^\infty - s\int_0^\infty e^{-st}\cos t\ dt\right) = s - s^2F(s)$$

従って、$F(s) = \dfrac{s}{s^2+1}$

(3)

$$\mathscr{L}(f) = \int_0^\infty e^{-st}2t\ dt = \left[-\frac{2}{s}e^{-st}t\right]_0^\infty + \frac{2}{s}\int_0^\infty e^{-st}\ dt = \frac{2}{s}\left[-\frac{1}{s}e^{-st}\right]_0^\infty = \frac{2}{s^2}$$

2. $Y(s) = \mathscr{L}(y)$ とすれば、両辺のラプラス変換により

$$\mathscr{L}(y' + 3y) = (s+3)Y(s) = 10\mathscr{L}(\sin t) = \frac{10}{s^2+1}.$$

$$Y(s) = \frac{10}{(s+3)(s^2+1)} = \frac{1}{s+3} + \frac{-s+3}{s^2+1}$$

より

$$y(t) = \mathscr{L}^{-1}(Y(s)) = e^{-3t} - \cos t + 3\sin t.$$

部分積分:

$F(x) = \displaystyle\int f(x)\ dx$ とするとき、

$$\int_a^b f(x)g(x)\ dt = \left[F(x)g(x)\right]_a^b - \int_a^b F(x)g'(x)\ dx$$