応用数学 II No.5 解答

1. (1) $\mathscr{L}(1) = \dfrac{1}{s}, \quad \mathscr{L}(\sin\omega t) = \dfrac{\omega}{s^2+\omega^2}$ より $\quad \mathscr{L}(1 * \sin\omega t) = \dfrac{\omega}{s(s^2+\omega^2)}$

(2) $\mathscr{L}(t) = \dfrac{1}{s^2}, \quad \mathscr{L}(e^t) = \dfrac{1}{s-1}$ より $\quad \mathscr{L}(t * e^t) = \dfrac{1}{s^2(s-1)}$

2. (1) $\dfrac{1}{s^2(s-1)} = \dfrac{1}{s^2} \times \dfrac{1}{s-1}, \quad \mathscr{L}^{-1}\left(\dfrac{1}{s^2}\right) = t, \quad \mathscr{L}^{-1}\left(\dfrac{1}{s-1}\right) = e^t$ より

$$\mathscr{L}^{-1}\left(\frac{1}{s^2(s-1)}\right) = t * e^t = \int_0^t \tau\ e^{t-\tau}\ d\tau = e^t\left\{[-\tau e^{-\tau}]_0^t + \int_0^t\ e^{-\tau}\ d\tau\right\}$$

$$= e^t(-te^{-t} + [-e^{-\tau}]_0^t) = -t - 1 + e^t$$

(2) $\dfrac{s^2}{(s^2+\omega^2)^2} = \dfrac{s}{s^2+\omega^2} \times \dfrac{s}{s^2+\omega^2}, \quad \mathscr{L}^{-1}\left(\dfrac{s}{s^2+\omega^2}\right) = \cos\omega t$

$$\mathscr{L}^{-1}\left(\frac{s^2}{(s^2+\omega^2)^2}\right) = \cos\omega t * \cos\omega t = \int_0^t\ \cos\omega\tau \times \cos\omega(t-\tau)d\tau$$

$$= \cos\omega t\int_0^t\ \cos^2\omega\tau\ d\tau + \sin\omega t\int_0^t\ \cos\omega\tau\ \ \sin\omega\tau\ d\tau$$

$$= \cos\omega t\int_0^t\ \frac{1}{2}(\cos 2\omega\tau + 1)\ d\tau + \sin\omega t\int_0^t\ \frac{1}{2}\sin 2\omega\tau\ d\tau$$

$$= \frac{1}{2}\cos\omega t\left[\frac{1}{2\omega}\sin 2\omega\tau + \tau\right]_0^t + \frac{1}{2}\sin\omega t\left[-\frac{1}{2\omega}\cos 2\omega\tau\right]_0^t$$

$$= \frac{1}{2}\cos\omega t\left(\frac{1}{2\omega}\sin 2\omega t + t\right) + \frac{1}{2}\sin\omega t\left(\frac{1}{2\omega} - \frac{1}{2\omega}\cos 2\omega t\right)$$

$$= \frac{1}{4\omega}\left\{2\omega t\cos\omega t + \cos\omega t\sin 2\omega t - \sin\omega t\cos 2\omega t + \sin\omega t\right\}$$

$$= \frac{1}{4\omega}\left\{2\omega t\cos\omega t + 2\sin\omega t\right\}$$

3. $\mathscr{L}(y(t)) = Y(s)$ とする。

$$Y(s) = \mathscr{L}(1) - \mathscr{L}(t * y(t)) = \frac{1}{s} - \mathscr{L}(t)\ Y = \frac{1}{s} - \frac{1}{s^2}\ Y(s)$$

$$\left(1 + \frac{1}{s^2}\right)\ Y(s) = \frac{s^2+1}{s^2}\ Y(s) = \frac{1}{s}$$

$Y(s) = \dfrac{s}{s^2+1}$ より

$$y(t) = \mathscr{L}^{-1}(Y(s)) = \cos t.$$

4 (1) 部分積分により

$$\int_0^1 xe^x\ dx = [xe^x]_0^1 - \int_0^1 e^x\ dx = e - [e^x]_0^1 = 1.$$

(2) $z = 2x - 3$ とするとき、$x = (z+3)/2$ より置換積分から

$$\int_1^2 (2x-3)^4\ dx = \int_{-1}^1 z^4 \frac{dx}{dz} dz = \frac{1}{2}\left[\frac{z^5}{5}\right]_{-1}^1 = \frac{1}{5}.$$

(3) $\dfrac{1}{2x^2 - 5x + 2} = \dfrac{1}{(2x-1)(x-2)} = \dfrac{1}{3}\left(\dfrac{1}{x-2} - \dfrac{2}{2x-1}\right)$ より

$$\int_{-1}^0 \frac{1}{2x^2-5x+2}\ dx = \frac{1}{3}\int_{-1}^0 \left(\frac{1}{x-2} - \frac{2}{2x-1}\right)\ dx = \frac{1}{3}\left[\log\left|\frac{x-2}{2x-1}\right|\right]_{-1}^0 = \frac{1}{3}\log 2.$$

予備問題：

1. (1) $y(t) = 2e^t - t - 1$, (2) $y(t) = e^{2t}$