応用数学 II No.8 解答

1. (1) 省略

(2)

$$a_0 = \frac{1}{2}\int_{-1}^{1} f(x)\ dx = \frac{1}{2}\int_{0}^{1} x\ dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 = \frac{1}{4}$$

$$a_n = \int_{-1}^{1} f(x)\ \cos n\pi x\ dx = \int_0^1 x\ \cos n\pi x\ dx = \left[x\ \frac{\sin n\pi x}{n\pi}\right]_0^1 - \frac{1}{n\pi}\int_0^1 \sin n\pi x\ dx$$

$$= \frac{1}{(n\pi)^2}[\cos n\pi x]_0^1 = \frac{1}{(n\pi)^2}((-1)^n - 1)$$

$$a_{2m} = 0, \qquad a_{2m+1} = -\frac{2}{((2m+1)\pi)^2}$$

$$b_n = \int_{-1}^{1} f(x)\ \sin n\pi x\ dx = \int_0^1 x\ \sin\ n\pi x\ dx = \left[-x\ \frac{\cos n\pi x}{n\pi}\right]_0^1 + \frac{1}{n\pi}\int_0^1 \cos n\pi x\ dx$$

$$= -\frac{(-1)^n}{n\pi} + \frac{1}{(n\pi)^2}[\sin n\pi x]_0^1 = -\frac{(-1)^n}{n\pi}$$

$$f(x) = \frac{\pi}{4} - 2\sum_{m=0}^{\infty} \frac{\cos(2m+1)\pi x}{((2m+1)\pi)^2} - \sum_{n=1}^{\infty} \frac{(-1)^n \sin n\pi x}{n\pi}$$

2. (1) 図省略。

(2) $-\pi/2 < x < -\pi$ について、$f(x) = -x - \pi$ として、周期的に拡張すれば、$f(x)$ は奇関数となる。したがって、もとの関数はこの一部であるから、 $a_n = 0.$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\ \sin nx\ dx = \frac{2}{\pi}\left\{\int_0^{\frac{\pi}{2}} x\ \sin nx\ dx + \int_{\frac{\pi}{2}}^{\pi} (\pi - x)\sin nx\ dx\right\}$$

$$= \frac{2}{\pi}\left\{-\left[x\ \frac{\cos nx}{n}\right]_0^{\frac{\pi}{2}} + \frac{1}{n}\int_0^{\frac{\pi}{2}} \cos nx\ dx + \left[-(\pi - x)\frac{\cos nx}{n}\right]_{\frac{\pi}{2}}^{\pi} - \frac{1}{n}\int_{\frac{\pi}{2}}^{\pi} \cos nx\ dx\right\}$$

$$= \frac{2}{\pi}\left\{-\frac{\pi}{2n}\cos\frac{\pi}{2} + \frac{1}{n}\left[\frac{\sin nx}{n}\right]_0^{\frac{\pi}{2}} + \frac{\pi}{2n}\cos\frac{\pi}{2} - \frac{1}{n}\left[\frac{\sin nx}{n}\right]_{\frac{\pi}{2}}^{\pi}\right\} = \frac{4}{\pi n^2}\ \sin\frac{n\pi}{2}$$

$$b_{2m} = 0, \quad b_{2m+1} = (-1)^m\ \frac{4}{\pi(2m+1)^2}$$

$$f(x) = \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^2} \sin(2m+1)x$$

3. (1) 図省略。$f(x)$ は偶関数である。

(2) (1) より $b_n = 0$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ dx = \frac{1}{\pi}\int_0^{\pi} 2x = \left[\frac{x^2}{\pi}\right]_0^{\pi} = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \cos nx\ dx = \frac{4}{\pi}\int_0^{\pi} x\ \cos nx\ dx = \frac{4}{\pi}\left\{\left[\frac{x \sin n\pi x}{n\pi}\right]_0^{\pi} - \frac{1}{n\pi}\int \sin n\pi x\ dx\right\}$$

$$= -\frac{4}{n\pi^2}\left[-\frac{\cos nx}{n\pi}\right]_0^{\pi} = \frac{4}{n^2\pi^3}((-1)^n - 1)$$

$$a_{2m} = 0, \quad a_{2m+1} = -\frac{8}{n^2\pi^3}$$

$$f(x) = \pi - \frac{8}{\pi^3} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \cos(2m+1)\pi x$$