

## 応用数学 II No.10 解答

1.

$$\begin{aligned} A(\omega) &= \frac{2}{\pi} \int_0^\infty f(v) \cos \omega v \, dv = \frac{2}{\pi} \int_0^a v \cos \omega v \, dv \\ &= \frac{2}{\pi} \left\{ \left[ \frac{v \sin \omega v}{\omega} \right]_0^a - \frac{1}{\omega} \int_0^a \sin \omega v \, dv \right\} = \frac{2}{\pi} \left\{ \frac{a}{\omega} \sin a\omega + \frac{1}{\omega} \left[ \frac{\cos \omega v}{\omega} \right]_0^a \right\} \\ &= \frac{2}{\pi \omega^2} \{ a\omega \sin a\omega + \cos a\omega - 1 \} \\ f(x) &= \frac{2}{\pi} \int_0^\infty \frac{a\omega \sin a\omega + \cos a\omega - 1}{\omega^2} \cos \omega x \, d\omega \end{aligned}$$

2.

$$\begin{aligned} B(\omega) &= \frac{2}{\pi} \int_0^\infty f(v) \sin \omega v \, dv = \frac{2}{\pi} \int_0^a \sin \omega v \, dv = \frac{2}{\pi} \left[ -\frac{\cos \omega v}{\omega} \right]_0^a \\ &= \frac{2}{\pi \omega} (1 - \cos a\omega) \\ f(x) &= \frac{2}{\pi} \int_0^\infty \frac{1 - \cos a\omega}{\omega} \sin \omega x \, d\omega \end{aligned}$$

**予備問題：**

1.  $f(x) = \pi/2$  ( $0 < x < \pi$ ),  $f(x) = 0$  ( $\pi < x$ ) とする。フーリエ正弦積分表示より、

$$\begin{aligned} B(\omega) &= \frac{2}{\pi} \int_0^\pi \frac{\pi}{2} \sin \omega v \, dv = \int_0^\pi \sin \omega v \, dv = \left[ -\frac{\cos \omega v}{\omega} \right]_0^\pi \\ &= \frac{1 - \cos \pi \omega}{\omega} \\ f(x) &= \int_0^\infty \frac{1 - \cos \pi \omega}{\omega} \sin x\omega \, d\omega \end{aligned}$$