# 応用数学 II No.11 解答

1. (1) 省略.

(2)

$$
\begin{aligned}
\hat{f}_c(\omega) &= \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\, \cos\omega x\, dx = \sqrt{\frac{2}{\pi}} \left\{ \int_0^1 \cos\omega x\, dx - \int_1^2 \cos\omega x\, dx \right\} \\
&= \sqrt{\frac{2}{\pi}} \left\{ \left[ \frac{\sin\omega x}{\omega} \right]_0^1 - \left[ \frac{\sin\omega x}{\omega} \right]_1^2 \right\} = \frac{1}{\omega} \sqrt{\frac{2}{\pi}} \{ \sin\omega - \sin 2\omega + \sin\omega \} \\
&= \frac{2}{\omega} \sqrt{\frac{2}{\pi}} \sin\omega(1 - \cos\omega).
\end{aligned}
$$

2. (1)　部分積分により

$$
\begin{aligned}
\mathscr{F}_s(e^{-ax}) &= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin\omega x\, dx = \sqrt{\frac{2}{\pi}} \left\{ \left[ -\frac{1}{a} e^{-ax} \sin\omega x \right]_0^\infty + \frac{1}{a} \int_0^\infty e^{-ax}\, \omega \cos\omega x\, dx \right\} \\
&= \sqrt{\frac{2}{\pi}} \left\{ \frac{\omega}{a} \left[ -\frac{1}{a} e^{-ax} \cos\omega x \right]_0^\infty - \frac{\omega^2}{a^2} \int_0^\infty e^{-ax}\, \sin\omega x\, dx \right\} \\
&= \sqrt{\frac{2}{\pi}}\, \frac{\omega}{a^2} - \frac{\omega^2}{a^2}\, \mathscr{F}_s(e^{-ax})
\end{aligned}
$$

$$
\left(1 + \frac{\omega^2}{a^2}\right) \mathscr{F}_s(e^{-ax}) = \sqrt{\frac{2}{\pi}}\, \frac{\omega}{a^2} \text{ より } \mathscr{F}_s(e^{-ax}) = \sqrt{\frac{2}{\pi}}\, \frac{\omega}{a^2 + \omega^2}
$$

(2)

$$
\mathscr{F}_s\left\{ f''(x) \right\} = \mathscr{F}_s\left\{ a^2 e^{-ax} \right\} = a^2 \mathscr{F}_s\left\{ e^{-ax} \right\} = -\omega^2 \mathscr{F}_s\left\{ e^{-ax} \right\} + \sqrt{\frac{2}{\pi}} \omega.
$$

$$
(a^2 + \omega^2) \mathscr{F}_s\left\{ e^{-ax} \right\} = \sqrt{\frac{2}{\pi}} \omega \text{ より } \mathscr{F}_s\left\{ e^{-ax} \right\} = \sqrt{\frac{2}{\pi}}\, \frac{\omega}{a^2 + \omega^2}.
$$