数学の考え方 No.8 解答

1. (1) $\lim_{n\to\infty}\dfrac{2-n}{3n-5}=\lim_{n\to\infty}\dfrac{\frac{2}{n}-1}{3-\frac{5}{n}}=-\dfrac{1}{3}$

(2) $\lim_{n\to\infty}\dfrac{1+2+\cdots+n}{n^2}=\lim_{n\to\infty}\dfrac{\frac{n(n+1)}{2}}{n^2}=\lim_{n\to\infty}\dfrac{1}{2}(1+\dfrac{1}{n})=\dfrac{1}{2}$

(3) $a_n=\dfrac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+2}-\sqrt{n-1}}=\dfrac{(n+1-n)(\sqrt{n+2}+\sqrt{n-1})}{(n+2-n+1)(\sqrt{n+1}+\sqrt{n})}=\dfrac{\sqrt{n+2}+\sqrt{n-1}}{3(\sqrt{n+1}+\sqrt{n})}$
より、

$\lim_{n\to\infty}a_n=\dfrac{\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{1}{n}}}{3(\sqrt{1+\frac{1}{n}}+1)}=\dfrac{1}{3}$

(4) $\dfrac{1}{(2n-1)(2n+1)}=\dfrac{1}{2}\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right)$ より

$a_n=\dfrac{1}{2}\left(\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\cdots+\dfrac{1}{(2n-1)(2n+1)}\right)=\dfrac{1}{2}\left(1-\dfrac{1}{2n+1}\right)$. $\lim_{n\to\infty}a_n=\dfrac{1}{2}$

発展問題

(1)

$$\begin{aligned}S_k &=\sum_{n=1}^{k}\frac{1}{n^2+3n+2}=\sum_{n=1}^{k}\frac{1}{(n+1)(n+2)}=\sum_{n=1}^{k}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\\ &=\left\{\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{k+1}-\frac{1}{k+2}\right\}=\frac{1}{2}-\frac{1}{k+2}.\end{aligned}$$

従って、

$$\sum_{n=1}^{\infty}\frac{1}{n^2+3n+2}=\lim_{k\to\infty}S_k=\lim_{k\to\infty}\left(\frac{1}{2}-\frac{1}{k+2}\right)=\frac{1}{2}.$$

(2) 無限級数の一般項は $\dfrac{(\sqrt{2}-1)^n}{2^{n-1}}$ であるから、

$$与式=2\sum_{n=1}^{\infty}\left(\frac{\sqrt{2}-1}{2}\right)^n=2\frac{\frac{\sqrt{2}-1}{2}}{1-\frac{\sqrt{2}-1}{2}}=\frac{2(\sqrt{2}-1)}{3-\sqrt{2}}=2(\sqrt{2}+1).$$