線形代数 No. 2 解答

1.

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{pmatrix} \overset{②-2\times①}{\longrightarrow} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 1 \end{pmatrix} \overset{①+②}{\longrightarrow} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & -2 & -2 & 1 \end{pmatrix}$$

$$\overset{-\frac{1}{2}\times②}{\longrightarrow} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -\frac{1}{2} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix}$$

2. (1)

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & 2 & 3 & 0 & 1 & 0 \\ -2 & -2 & 0 & 0 & 0 & 1 \end{pmatrix} \overset{\substack{②-2\times① \\ ③+①}}{\longrightarrow} \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\overset{②\leftrightarrow③}{\longrightarrow} \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{pmatrix} \overset{①+②}{\longrightarrow} \begin{pmatrix} 2 & 0 & 2 & 2 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{pmatrix}$$

$$\overset{\substack{①-2\times③ \\ ②-③}}{\longrightarrow} \begin{pmatrix} 2 & 0 & 0 & 6 & -2 & 1 \\ 0 & -1 & 0 & 3 & -1 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{pmatrix} \overset{\substack{\frac{1}{2}\times① \\ -②}}{\longrightarrow} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{pmatrix}$$

従って、逆行列は $\begin{pmatrix} 3 & -1 & \frac{1}{2} \\ -3 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$.

(2)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & \frac{1}{2} \\ -3 & 1 & -1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ -4 \\ -3 \end{pmatrix}$$