

数学解析 I No.8 解答

1. (1)

$$\int \left(\frac{2}{1+x^2} + 3x \right) dx = 2 \int \frac{1}{1+x^2} dx + 3 \int x dx = 2 \tan^{-1} x + \frac{3}{2} x^2 + C.$$

(2) $t = \cos x$ とすると、 $\frac{dx}{dt} = \frac{1}{\frac{dt}{dx}} = \frac{1}{-\sin x}$. 従って、

$$\int 3 \cos^2 x \sin x dx = 3 \int t^2 \sin x \frac{1}{-\sin x} dt = -3 \int t^2 dt = -t^3 + C = -\cos^3 x + C.$$

(3) $t = x + \frac{1}{2}$ とすると、 $\frac{dx}{dt} = 1$. 置換積分から

$$\begin{aligned} \int \frac{2}{x^2 + x + 1} dx &= 2 \int \frac{1}{(x + \frac{1}{2})^2 + \frac{3}{4}} dx = 2 \int \frac{1}{t^2 + (\frac{\sqrt{3}}{2})^2} dt = \frac{4}{\sqrt{3}} \tan^{-1} \frac{2}{\sqrt{3}} t + C \\ &= \frac{4}{\sqrt{3}} \tan^{-1} \frac{(2x+1)}{\sqrt{3}} + C. \end{aligned}$$

(4) 部分積分より

$$\begin{aligned} \int \tan^{-1} x dx &= \int 1 \cdot \tan^{-1} x dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx \\ &= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + C. \end{aligned}$$