# Analysis II　No. 2　解答

1. (1) $\dfrac{\partial z}{\partial x} = 3x^2 - 8xy + y,\ \dfrac{\partial z}{\partial y} = -4x^2 + x + 6y.$

(2) $z = \log_y x = \dfrac{\log x}{\log y}$ であるから、

$$\frac{\partial z}{\partial x} = \frac{1}{x \log y}, \quad \frac{\partial z}{\partial y} = -\frac{\log x}{y(\log y)^2}.$$

(3) $\dfrac{\partial z}{\partial x} = \dfrac{x}{\sqrt{x^2 - 4y^2}}, \quad \dfrac{\partial z}{\partial y} = \dfrac{-4y}{\sqrt{x^2 - 4y^2}}$

(4)

$$\frac{\partial z}{\partial x} = \frac{1}{1 + \left(\frac{x+y}{x-y}\right)^2} \frac{-2y}{(x-y)^2} = \frac{-y}{x^2 + y^2}, \quad \frac{\partial z}{\partial y} = \frac{1}{1 + \left(\frac{x+y}{x-y}\right)^2} \frac{2x}{(x-y)^2} = \frac{x}{x^2 + y^2}.$$

2. 点 $(a, b, f(a, b))$ での接平面の方程式は $z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$ である。$f_x = 2x,\ f_y = 2y$ より、$f_x(1, 2) = 2,\ f_y(1, 2) = 4.$ したがって、$z - 5 = 2(x - 1) + 4(y - 2).$