Analysis II No. 3 解答

1. $\dfrac{dz}{dt} = \dfrac{dx}{dt}\dfrac{\partial z}{\partial x} + \dfrac{dy}{dt}\dfrac{\partial z}{\partial y}$

(1) $\dfrac{dz}{dt} = 10\sin t\cos t = 5\sin 2t.$

(2) $\dfrac{dz}{dt} = e^t\cos(e^t)\cos(\log t) - \dfrac{1}{t}\sin(e^t)\sin(\log t).$

2. $\dfrac{\partial z}{\partial u} = \dfrac{\partial x}{\partial u}\dfrac{\partial z}{\partial x} + \dfrac{\partial y}{\partial u}\dfrac{\partial z}{\partial y}$, $\dfrac{\partial z}{\partial v} = \dfrac{\partial x}{\partial v}\dfrac{\partial z}{\partial x} + \dfrac{\partial y}{\partial v}\dfrac{\partial z}{\partial y}$

(1) $z_u = e^x(\cos y - \sin y)$, $z_v = -e^x(\sin y + \cos y)$.

(2) $z_u = -\dfrac{2(u^2+v^2)}{u^2v}\left(\sin\dfrac{v}{u} + \cos\dfrac{u}{v}\right)\left(\cos\dfrac{v}{u} + \sin\dfrac{u}{v}\right)$,
$z_v = \dfrac{2(u^2+v^2)}{uv^2}\left(\sin\dfrac{v}{u} + \cos\dfrac{u}{v}\right)\left(\cos\dfrac{v}{u} + \sin\dfrac{u}{v}\right)$.

3. (1) $z_x = 2x + 2y$, $z_y = 2x + 6y$, $z_{xx} = 2$, $z_{xy} = z_{yx} = 2$, $z_{yy} = 6$.

(2) $z_x = -\dfrac{y}{x^2}e^{\frac{y}{x}}$, $z_y = \dfrac{1}{x}e^{\frac{y}{x}}$, $z_{xx} = \dfrac{y^2-2xy}{x^4}e^{\frac{y}{x}}$, $z_{xy} = z_{yx} = -\dfrac{x+y}{x^3}e^{\frac{y}{x}}$, $z_{yy} = \dfrac{1}{x^2}e^{\frac{y}{x}}$