

数学解析 II No. 8 解答

1.

$$(1) \int_{-1}^1 \int_2^4 (x+y) \, dx dy = \int_{-1}^1 \left[\frac{x^2}{2} + xy \right]_2^4 dy = \int_{-1}^1 (6+2y) \, dy = 12$$

$$(2) \int_1^3 \int_1^{x^2} \frac{x}{y^2} \, dy dx = \int_1^3 x \int_1^{x^2} \frac{1}{y^2} \, dy dx = \int_1^3 x \left[-\frac{1}{y} \right]_1^{x^2} dx = \int_1^3 x \left(-\frac{1}{x^2} + 1 \right) dx$$

$$= \int_1^3 \left(x - \frac{1}{x} \right) dx = \left[\frac{x^2}{2} - \log x \right]_1^3 = 4 - \log 3.$$

$$(3) x^2 + y^2 = t \text{ とすれば、 } \frac{dx}{dt} = \frac{1}{2x}.$$

$$\int_0^1 \int_0^{\sqrt{1-y^2}} x \sqrt{x^2 + y^2} \, dx dy = \int_0^1 \int_{y^2}^1 x \sqrt{t} \frac{1}{2x} \, dt dy = \int_0^1 \int_{y^2}^1 \frac{\sqrt{t}}{2} \, dt dy = \frac{1}{2} \int_0^1 \left[\frac{t^{\frac{3}{2}}}{1 + \frac{1}{2}} \right]_{y^2}^1 dy$$

$$= \frac{1}{3} \int_0^1 (1 - y^3) dy = \frac{1}{3} \left[y - \frac{y^4}{4} \right]_0^1 = \frac{1}{4}.$$

2.

$$\int \int_D x^2 y^2 (x^2 - y^3) \, dx dy = \int_0^1 \int_{-1}^2 (x^4 y^2 - x^2 y^5) \, dy dx = \int_0^1 \left[\frac{x^4 y^3}{3} - \frac{x^2 y^6}{6} \right]_{-1}^2 dy$$

$$= \int_0^1 \left(3x^4 - \frac{63x^2}{6} \right) dx = \left[\frac{3x^4}{5} - \frac{63x^3}{18} \right]_0^1 = -\frac{29}{10}.$$

発展問題：積分領域は $\{(x, y) | 1 < x < e, 0 < y < \log x\} = \{(x, y) | 0 < y < 1, e^y < x < e\}$ で与えられる。

$$\int_1^e \int_0^{\log x} \frac{1+y}{x} \, dy dx = \int_0^1 \int_{e^y}^e \frac{1+y}{x} \, dx dy = \int_0^1 (1+y) [\log x]_{e^y}^e dy = \int_0^1 (1+y)(1-y) \, dy$$

$$= \left[y - \frac{y^3}{3} \right]_0^1 = \frac{2}{3}.$$