数学解析 II No. 9 解答

1. (1)

$$\int\int_D (x+y^2)\ dxdy = \int_0^1\int_0^1 (x+y^2)\ dxdy = \int_0^1 \left[\frac{x^2}{2}+y^2x\right]_0^1 = \left[\frac{y}{2}+\frac{y^3}{3}\right]_0^1 = \frac{5}{6}$$

(2) $D=\{(x,y)|\ 0\le x\le \frac{\pi}{2},\ 0\le y\le \frac{\pi}{2}-x\}$. $x+y=z$ とするとき、

$$\int\int_D \sin(x+y)\ dxdy = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}-x} \sin(x+y)\ dydx = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin z\frac{dy}{dz}dz$$

$$= \int_0^{\frac{\pi}{2}} [-\cos z]_x^{\frac{\pi}{2}}\ dx = \int_0^{\frac{\pi}{2}}\left(\cos x-\cos\frac{\pi}{2}\right)\ dx = [\sin x]_0^{\frac{\pi}{2}} = 1.$$

(3)

$$\int\int_D \sqrt{1-x^2}\ dxdy = \int_0^1\int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\ dydx = \int_0^1 \sqrt{1-x^2}\,[y]_0^{\sqrt{1-x^2}}\ dx = \int_0^1 (1-x^2)\ dx$$

$$= \left[x-\frac{x^3}{3}\right]_0^1 = \frac{2}{3}.$$

(4)

$$\int\int_D \frac{1}{\sqrt{-y+\cos^2 x}}\ dxdy = \int_0^{\frac{\pi}{2}}\int_0^{\cos^2 x} \frac{1}{\sqrt{-y+\cos^2 x}}\ dydx = \int_0^{\frac{\pi}{2}} \left[-\frac{\sqrt{-y+\cos^2 x}}{1-\frac{1}{2}}\right]_0^{\cos^2 x}\ dx$$

$$= 2\int_0^{\frac{\pi}{2}} |\cos x|\ dx = 2[\sin x]_0^{\frac{\pi}{2}} = 2.$$

発展問題 (1) $1+x+y=t$ とする。

$$I_n = \int_0^n\int_0^{n-x} \frac{1-x-y}{(1+x+y)^4}\ dydx = \int_0^n\int_{1+x}^{1+n} \frac{2-t}{t^4}\ dtdx = \int_0^n \left[-\frac{2}{3t^3}+\frac{1}{2t^3}\right]_{1+x}^{1+n}\ dx$$

$$= \int_0^n \left(-\frac{2}{3(1+n)^3}+\frac{2}{3(1+x)^3}+\frac{1}{2(1+n)^2}-\frac{1}{2(1+x)^2}\right)\ dx$$

$$= \left[-\frac{2x}{3(1+n)^3}+\frac{x}{2(1+n)^2}-\frac{1}{3(1+x)^2}+\frac{1}{2(1+x)}\right]_0^n$$

$$= -\frac{2n}{3(1+n)^3}+\frac{n}{2(1+n)^2}-\frac{1}{3(1+n)^2}+\frac{1}{2(1+n)}-\frac{1}{6}.$$

(2)

$$\int\int_D \frac{1-x-y}{(1+x+y)^4}\ dxdy = \lim_{n\to\infty} I_n = -\frac{1}{6}.$$