Relativistic Impulse Approximation
Analysis of Unstable Nuclei:
Calcium and Nickel Isotopes

Kaori Kaki
Department of Physics, Shizuoka University

21 February 2009
Relativistic Impulse Approximation (RIA)

- optical potentials

Dirac equation for a projectile proton scattering from a target nucleus given by the optical model:

\[
\left[ p - m - \hat{U}(\mathbf{r}) \right] \psi(\mathbf{r}) = 0,
\]

\[
p = \gamma_\mu p^\mu, \quad p^\mu = (E, \mathbf{p})
\]

the momentum space Dirac equation

\[
\left( \gamma^0 E - \gamma^\alpha p^\alpha - m \right) \psi(\mathbf{p}') - \frac{1}{(2\pi)^3} \int d^3p \; \hat{U}(\mathbf{p}', \mathbf{p}) \psi(\mathbf{p}) = 0
\]
by relativistic analog of non-relativistic multiple scattering theory, optical potential in coordinate space:

\[
\hat{U}(\mathbf{r}) = \langle \Phi | \sum_i t_i | \Phi \rangle \quad \text{1st order term}
\]

\[
+ \sum_{i \neq j} \sum_j < \Phi | t_i \tilde{G} t_j | \Phi >
\]

\[
- \frac{A - 1}{A} \langle \Phi | \sum_i t_i | \Phi \rangle \tilde{G} < \Phi | \sum_j t_j | \Phi >
\]

2nd order term
the generalized RIA optical potential
in the momentum space for the 1st order term

\[ \hat{U}(p', p) = -\frac{1}{4} \text{Tr} \left\{ \int \frac{d^3k}{(2\pi)^3} \hat{M}_{pp}(p, k - \frac{q}{2} \rightarrow p', k + \frac{q}{2}) \hat{\rho}_p(k, q) \right\} \]

\[ -\frac{1}{4} \text{Tr} \left\{ \int \frac{d^3k}{(2\pi)^3} \hat{M}_{pn}(p, k - \frac{q}{2} \rightarrow p', k + \frac{q}{2}) \hat{\rho}_n(k, q) \right\} \]

optimal factorization: \( (k = 0) \)

\[ \hat{U}(p', p) = -\frac{1}{4} \text{Tr} \left\{ \hat{M}_{pp}(p, -\frac{q}{2} \rightarrow p', \frac{q}{2}) \hat{\rho}_p(q) \right\} \]

\[ -\frac{1}{4} \text{Tr} \left\{ \hat{M}_{pn}(p, -\frac{q}{2} \rightarrow p', \frac{q}{2}) \hat{\rho}_n(q) \right\} \]
density matrices

\[ \hat{\rho}(q) = \int d^3 r \ e^{iq \cdot r} \ \hat{\rho}(r) \]

\[ \hat{\rho}(k, q) = \int d^3 r \ e^{iq \cdot r} \ \hat{\rho}(r(k)) \]

each nuclear density distribution

\[ \hat{\rho}(r) = \rho_S(r) + \gamma^0 \rho_V(r) - \frac{i \alpha \cdot \hat{r}}{2} \rho_T(r) \]

\[ \uparrow \quad \uparrow \quad \uparrow \]

scalar vector tensor
the 2nd order optical potential in the momentum space

\[
\hat{U}(p', p) = \int \frac{d^3k}{(2\pi)^2} \frac{1}{4} \text{Tr}_2 \left\{ \hat{M}(k, -\frac{q_b}{2} \to p', \frac{q_b}{2}) \hat{\rho}(q_b) \right\} \\
\times \hat{C}(q_b, q_a) \overline{G}(k) \frac{1}{4} \text{Tr}_3 \left\{ \hat{M}(p, -\frac{q_a}{2} \to k, \frac{q_a}{2}) \hat{\rho}(q_a) \right\}
\]

\[\rho = \rho_0 + \sum_{\mu} C_{\mu}(p, q, l) \overline{G}(l, k, p, q) \rho_0 \overline{G}(k, q, l, p)\]

propagator

\[q_a = p - k\]

\[q_b = k - p'\]
propagator
\[
\widetilde{G}(k) = (k - m - \gamma^0 E_A + i\varepsilon)^{-1}
\]

correlation function
\[
\hat{\rho}(q_b) \hat{C}(q_b, q_a) \hat{\rho}(q_a)
\]
correlation function

\[ \hat{\rho}(q_b) \hat{C}(q_b, q_a) \hat{\rho}(q_a) \]

\[ = \int d^3r_a \int d^3r_b e^{i\mathbf{r}_a \cdot \mathbf{q}_a} e^{i\mathbf{r}_b \cdot \mathbf{q}_b} f(|\mathbf{r}_a - \mathbf{r}_b|) \hat{\rho}(\mathbf{r}_a) \hat{\rho}(\mathbf{r}_b) \]

\[ f(r) = \sum_{\alpha=1}^{3} f_\alpha e^{-\frac{r^2}{R^2_\alpha}} \]

the same parameters as in

density distributions for Ca isotopes

relativistic mean field theory (rmft)

for $^{60-74}$Ca

private communication with L.S.Geng in RCNP
density distributions for Ni isotopes

relativistic mean field theory (rmft)

TMA code: Y. Sugahara & H. Toki
NPA579 (1994) 557
Relativistic Impulse Approximation

\( {}^{40}\text{Ca} \) at \( E_p=200 \text{ MeV} \), \( E_p=300 \text{ MeV} \), \( E_p=400 \text{ MeV} \)

2nd

1st

med.

exp. data

from global optical potential fittings
Relativistic Impulse Approximation

$^{48}\text{Ca}$

- - - 2nd

1st

- - - med.

exp. data

A.E. Feldman et al.
G.W. Hoffman et al.
Relativistic Impulse Approximation

\( ^{58}\text{Ni} \)

- - - - 2nd
  _____ 1st
  - - - - med.

exp. data

H. Sakaguchi et al.
PRC57(1998)1749
Relativistic Impulse Approximation

- $^{60,68,74}\text{Ca}$ at 200 MeV
- $^{60,68,74}\text{Ca}$ at 300 MeV
- $^{60,68,74}\text{Ca}$ at 400 MeV
- $^{60,68,74}\text{Ca}$ at 500 MeV
Relativistic Impulse Approximation

- $^{48-64}$Ni at 200 MeV
- $^{48-64}$Ni at 300 MeV
- $^{48-64}$Ni at 400 MeV
- $^{48-64}$Ni at 500 MeV

- $^{66-82}$Ni at 200 MeV
- $^{66-82}$Ni at 300 MeV
- $^{66-82}$Ni at 400 MeV
- $^{66-82}$Ni at 500 MeV
potential depth

scalar potential

- $^{40}\text{Ca}$
- $^{60}\text{Ca}$
- $^{74}\text{Ca}$
potential depth

vector potential

$^{40}\text{Ca}$

$^{60}\text{Ca}$

$^{74}\text{Ca}$
relation between $A$ & $\Delta$

\[ \Delta = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} \]
reaction cross sections & 1\textsuperscript{st} dip position

Ca isotopes
reaction cross sections & 1st dip position

Ni isotopes

- 200 MeV
- 300
- 400
- 500

Ni isotopes
study for neutron distribution


\[
\rho(r) = \frac{\rho_0}{1 + \exp\left\{\left(\frac{r - r_0}{a}\right)\right\}}
\]

normalized by
\[
A - Z = 4\pi \int \rho(r) r^2 dr
\]

radial parameter

diffuseness parameter

*proton distributions are fixed to the charge or rmf density
Fourier transformation

\[ \rho(q) = 4\pi \int_0^\infty j_0(qr) \frac{\rho_0}{1 + e^{(r-r_0)/a}} r^2 \, dr \]

\[ \approx \frac{4\pi\rho_0}{q^3} \frac{\pi qa}{\sinh(\pi qa)} \]

\[ \times \left\{ \pi qa \cdot \coth(\pi qa) \sin(qr_0) - qr_0 \cdot \cos(qr_0) \right\} \]
differential cross section

\begin{equation}
\frac{d\sigma}{d\Omega} = |f_B(\theta)|^2 = \left(\frac{\mu}{2\pi\hbar}\right)^2 |t(q)|^2 \rho^2(q)
\end{equation}

1st dip position \iff \rho(q) = 0
mean-square radius

\[ \langle r^2 \rangle = 4\pi \int r^2 \rho(r) r^2 dr / 4\pi \int \rho(r) r^2 dr \]

\[ = \frac{1}{5} \left[ 7(\pi a)^2 + 3r_0^2 \right] \]

analytic function of the parameters
contour map of msr & dip with respect to $r_0$ & $\alpha$

$\langle r^2 \rangle$ (fm$^2$)

$\theta$ (deg)

analytic cal.
contour map of rcs & dip with respect to \( r_0 \) & \( \alpha \)

observables

\[
\sigma_r (\text{fm}^2)
\]

\[
\theta (\text{deg})
\]
to determine parameters

$^{60}$Ca rmft

$\sigma_r = 75.34 \ (fm^2)$

$\theta = 12.22 \ (deg.)$

$r_0 = 4.32 \ (fm)$

$a = 0.64 \ (fm)$
obtained density distribution for neutron
summary

◆ observables of proton-elastic scattering from $^{40,48,60-74}$Ca nuclei and $^{48-82}$Ni nuclei
  ● incident energies: 200, 300, 400 & 500 MeV
  ● Relativistic Impulse Approximation $\rightarrow$ IA2 parameters
  ● Relativistic Mean Field Theory $\rightarrow$ nuclear densities

◆ medium effects
  ● reaction cross section $\rightarrow$ a little bit smaller

◆ multiple scattering effect (2$^{\text{nd}}$ order potential)
  ● contributions $\rightarrow$ at rather low energy & larger angle
  ● reaction cross section $\rightarrow$ a little bit larger
  ● dip positions $\rightarrow$ slightly different but not significant
conclusion

- to determine the neutron distribution of unstable nuclei
  - RIA with IA2 parameter
  - $k=0$ for incident proton energies: 200-500 MeV
  - medium & multiple scattering effects: not significant role
    - both in reaction cross section and dip positions

near future

- target nucleus; Ni isotopes, Sn isotopes
  - energy range: 200-500 MeV
  - 1st order calculations of RIA with OF
  - contour maps for the WS parameters: $r_0$, $a$
Relativistic Impulse Approximation

$^{60,68,74}\text{Ca}$

200 MeV

---

2nd

1st

---

med.

$E_p = 200$ MeV
Relativistic Impulse Approximation

$^{60,68,74}\text{Ca}$

300 MeV

2nd
1st
med.

$E_p = 300$ MeV
Relativistic Impulse Approximation

$60, 68, 74 \text{Ca}$

400 MeV

---

2nd

1st

med.

$E_p = 400 \text{ MeV}$
Relativistic Impulse Approximation

$^{60,68,74}$Ca

500 MeV

- - - 2nd

1st

- - - med.

$E_p = 500$ MeV
Relativistic Impulse Approximation

48-64Ni

48Ni

52Ni

64Ni

\( E_p = 200 \text{ MeV} \)
Relativistic Impulse Approximation

48-64\textsuperscript{Ni}

- - - 2nd

1st

med.

$E_p = 300$ MeV
Relativistic Impulse Approximation

48-64Ni

$E_p = 400$ MeV
Relativistic Impulse Approximation

48-64\textsuperscript{Ni}

- --- 2nd
- 1st
- --- med.

$E_p = 500$ MeV
Relativistic Impulse Approximation

$^{70-82}$Ni

- 2nd
- 1st
- med.

$E_p = 200$ MeV
Relativistic Impulse Approximation

$^{70-82\text{Ni}}$

2nd
1st
med.

$E_p = 300$ MeV
Relativistic Impulse Approximation

$^{70-82}\text{Ni}$

- --- 2nd
- ---- 1st
- ------ med.

$E_p = 400$ MeV
Relativistic Impulse Approximation

$^{70-82}\text{Ni}$

- 2nd
- 1st
- med.

$E_p = 500$ MeV