Relativistic Impulse Approximation Analysis of Unstable Nuclei: Calcium and Nickel Isotopes

Kaori Kaki
Department of Physics, Shizuoka University

21 February 2009

Relativistic Impulse Approximation (RIA)

- optical potentials

Dirac equation for a projectile proton scattering from a target nucleus given by the optical model:

$$[\not{p} - m - \hat{U}(\mathbf{r})]\,\psi(\mathbf{r}) = 0,$$

$$\not{p} = \gamma_\mu p^\mu,\; p^\mu = (E, \mathbf{p})$$

the momentum space Dirac equation

$$\left(\gamma^0 E - \boldsymbol{\gamma}\cdot\mathbf{p}' - m\right)\psi(\mathbf{p}') - \frac{1}{\left(2\pi\right)^3}\int d^3p\,\hat{U}(\mathbf{p}',\mathbf{p})\,\psi(\mathbf{p}) = 0$$

by relativistic analog of non-relativistic multiple scattering theory, optical potential in coordinate space:

$$\hat{U}(\mathbf{r}) = \langle \Phi | \sum_i t_i | \Phi \rangle$$

← 1st order term

$$\left\{ \begin{array}{l} + \sum_{i \neq j} \sum_j \langle \Phi | t_i \bar{G} t_j | \Phi \rangle \\ - \dfrac{A-1}{A} \langle \Phi | \sum_i t_i | \Phi \rangle \bar{G} \langle \Phi | \sum_j t_j | \Phi \rangle \end{array} \right.$$

2nd order term

the generalized RIA optical potential

in the momentum space for the 1st order term

$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\int \frac{d^3k}{(2\pi)^3}\,\hat{M}_{pp}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',k+\frac{\mathbf{q}}{2})\;\hat{\rho}_p(\mathbf{k},\mathbf{q})\right\}$$

$$-\frac{1}{4}\mathrm{Tr}\left\{\int \frac{d^3k}{(2\pi)^3}\,\hat{M}_{pn}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',k+\frac{\mathbf{q}}{2})\;\hat{\rho}_n(\mathbf{k},\mathbf{q})\right\}$$

optimal factorization : $(\mathbf{k}=0)$

simple

t ρ -form

$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pp}(\mathbf{p},-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',\frac{\mathbf{q}}{2})\;\hat{\rho}_p(\mathbf{q})\right\}$$

$$-\frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pn}(\mathbf{p},-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',\frac{\mathbf{q}}{2})\;\hat{\rho}_n(\mathbf{q})\right\}$$

density matrices

$$\widehat{\rho}(\mathbf{q}) = \int d^3r \; e^{i\mathbf{q}\cdot\mathbf{r}} \; \widehat{\rho}(\mathbf{r})$$

$$\widehat{\rho}(\mathbf{k},\mathbf{q}) = \int d^3r \; e^{i\mathbf{q}\cdot\mathbf{r}} \; \widehat{\rho}(r(k))$$

each nuclear density distribution

$$\widehat{\rho}(\mathbf{r}) = \rho_S(\mathbf{r}) + \gamma^0 \rho_V(\mathbf{r}) - \frac{i\boldsymbol{\alpha}\cdot\widehat{\mathbf{r}}}{2}\rho_T(\mathbf{r})$$

scalar　　vector　　tensor

the 2nd order optical potential in the momentum space

$$\hat{U}(\mathbf{p}',\mathbf{p}) = \int \frac{d^3k}{(2\pi)^2} \frac{1}{4} \mathrm{Tr}_2 \left\{ \hat{M}(\mathbf{k}, -\frac{\mathbf{q}_b}{2} \rightarrow \mathbf{p}', \frac{\mathbf{q}_b}{2}) \hat{\rho}(\mathbf{q}_b) \right\}$$

$$\times \hat{C}(\mathbf{q}_b, \mathbf{q}_a) \bar{G}(\mathbf{k}) \frac{1}{4} \mathrm{Tr}_3 \left\{ \hat{M}(\mathbf{p}, -\frac{\mathbf{q}_a}{2} \rightarrow \mathbf{k}, \frac{\mathbf{q}_a}{2}) \hat{\rho}(\mathbf{q}_a) \right\}$$

propagator

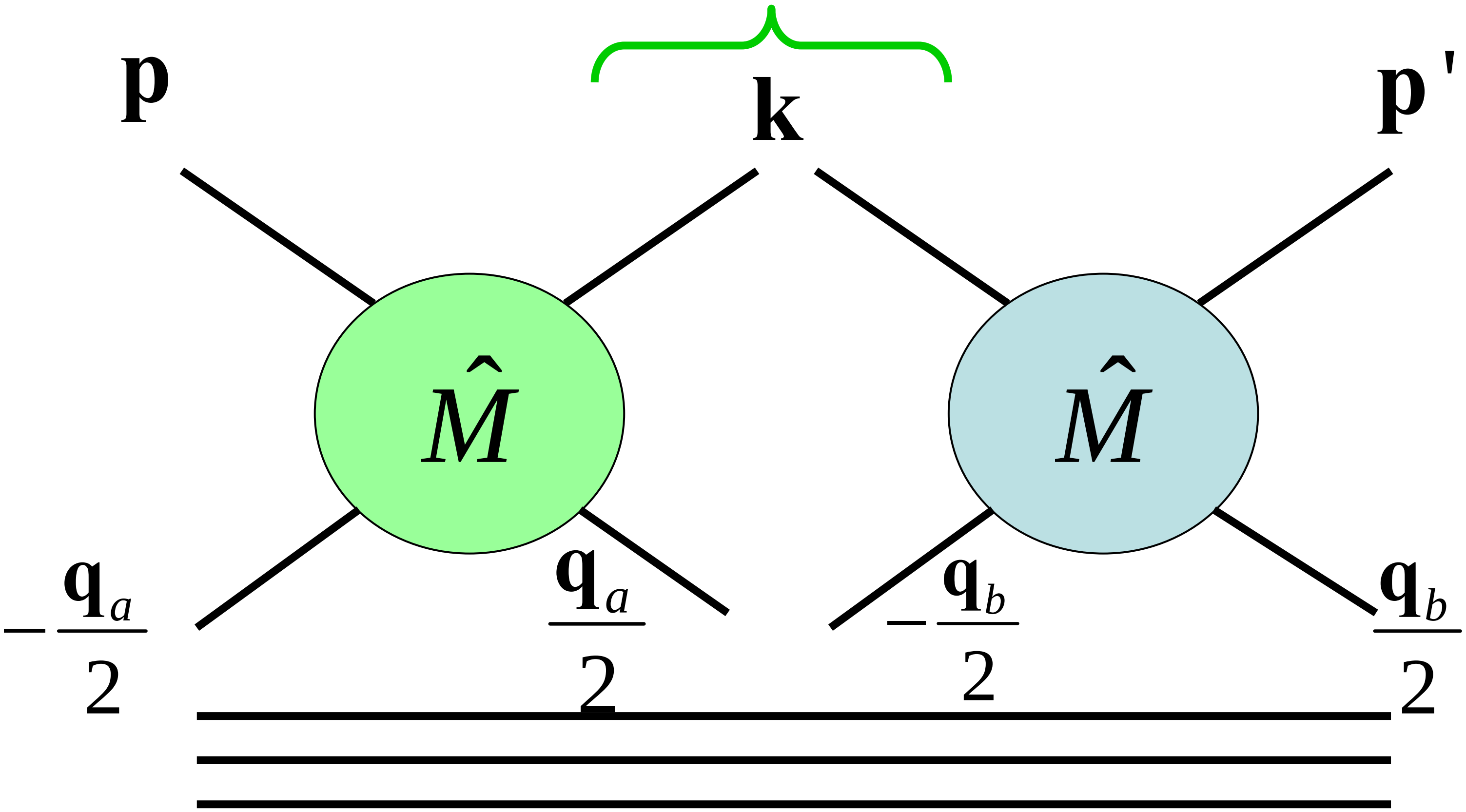
correlation part

$$\mathbf{q}_a = \mathbf{p} - \mathbf{k}$$

$$\mathbf{q}_b = \mathbf{k} - \mathbf{p}'$$

propagator

$$\bar{G}(\mathbf{k}) = (\not{k} - m - \gamma^0 \bar{E}_A + i\varepsilon)^{-1}$$



correlation function $\hat{\rho}(\mathbf{q}_b)\hat{C}(\mathbf{q}_b, \mathbf{q}_a)\hat{\rho}(\mathbf{q}_a)$

correlation function

$$\hat{\rho}(\mathbf{q}_b)\hat{C}(\mathbf{q}_b,\mathbf{q}_a)\hat{\rho}(\mathbf{q}_a)$$

$$=\int d^3 r_a \int d^3 r_b e^{i\mathbf{r}\cdot\mathbf{q}_a} e^{i\mathbf{r}\cdot\mathbf{q}_b} f(|\mathbf{r}_a-\mathbf{r}_b|)\hat{\rho}(\mathbf{r}_a)\hat{\rho}(\mathbf{r}_b)$$

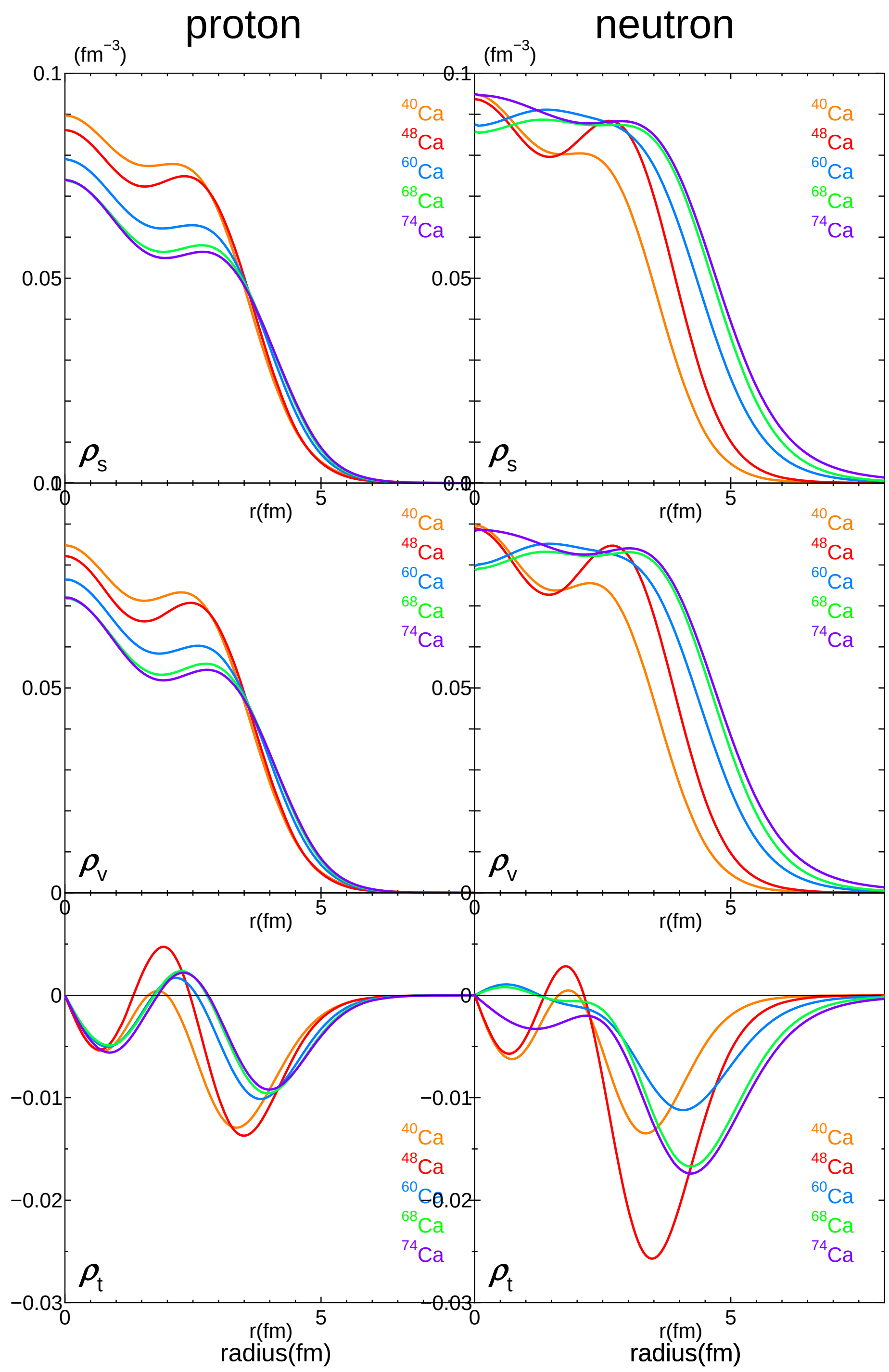
$$f(r)=\sum_{\alpha=1}^{3} f_\alpha e^{-\frac{r^2}{R^2{}_\alpha}}$$

the same parameters as in

J.D.Lumpe & L.Ray, Phys.Rev.C35(1987)1040 density distributions for Ca isotopes

relativistic mean field theory (rmft)

for $^{60\text{-}74}$Ca
private communication with L.S.Geng in RCNP



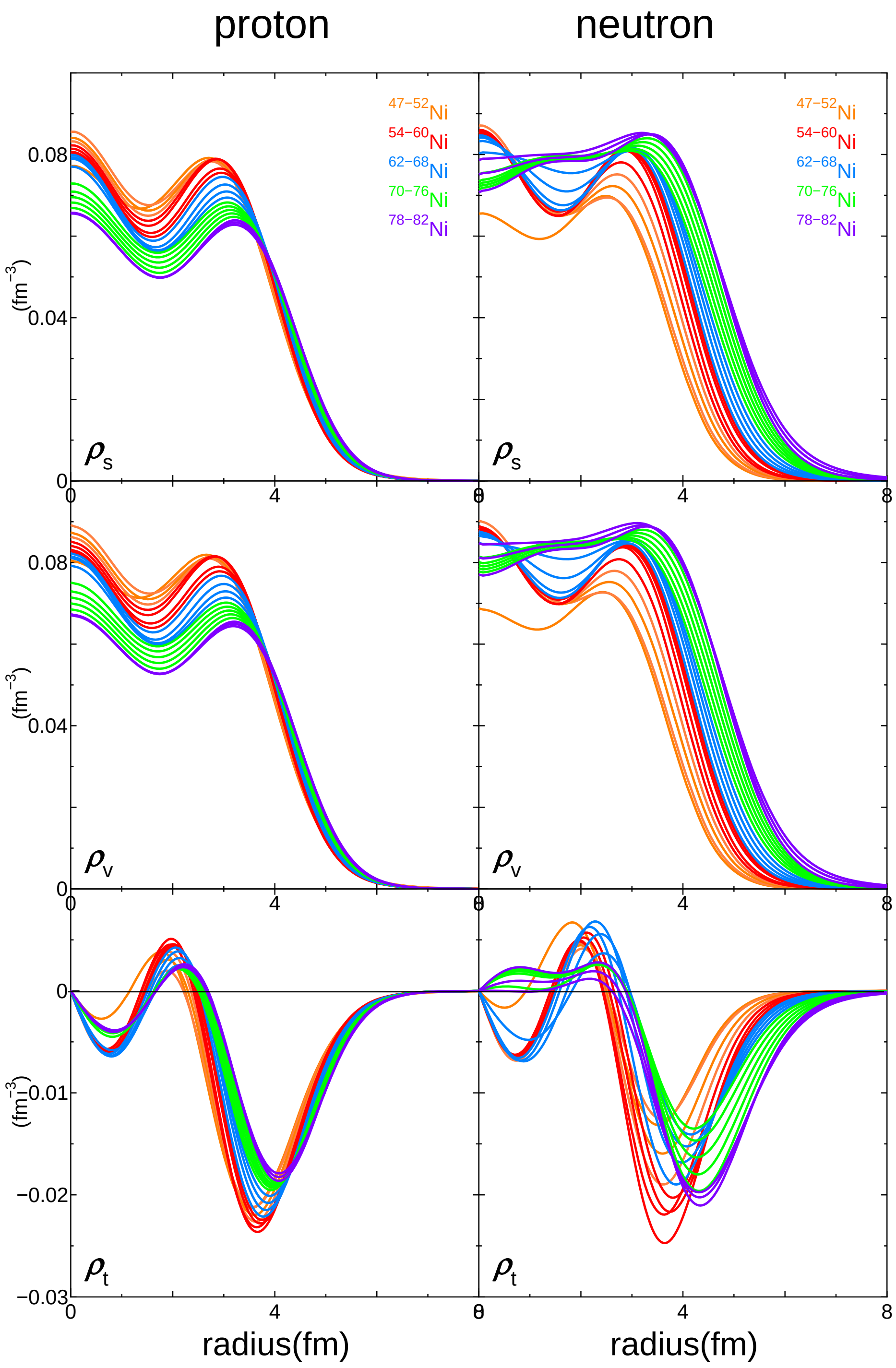
density distributions for Ni isotopes

⬆

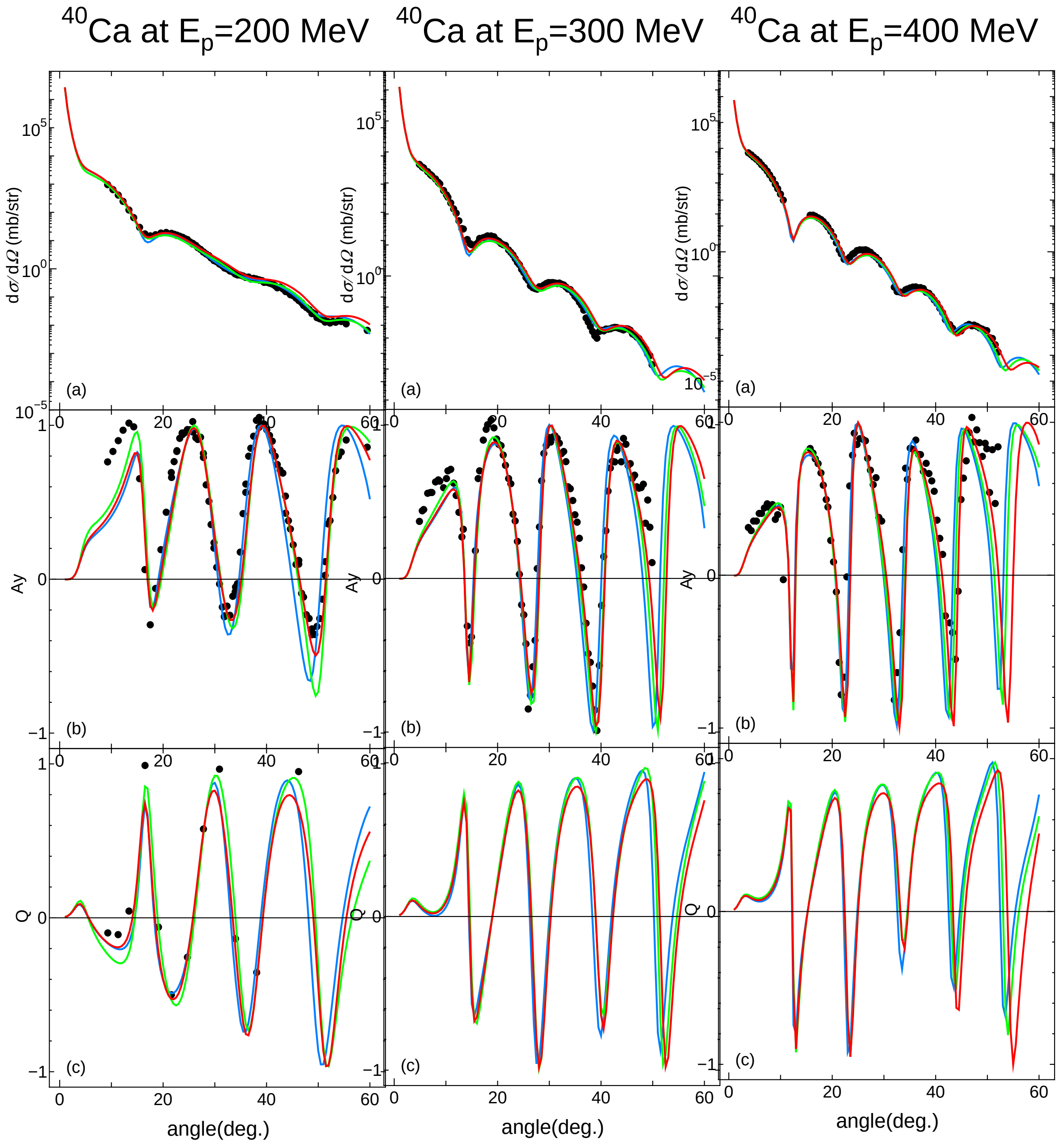
relativistic mean field theory (rmft)

TMA code :Y.Sugahara & H.Toki

NPA579 (1994) 557



Relativistic Impulse Approximation



^{40}Ca

- 2nd
- 1st
- med.

exp. data

from global optical potential fittings

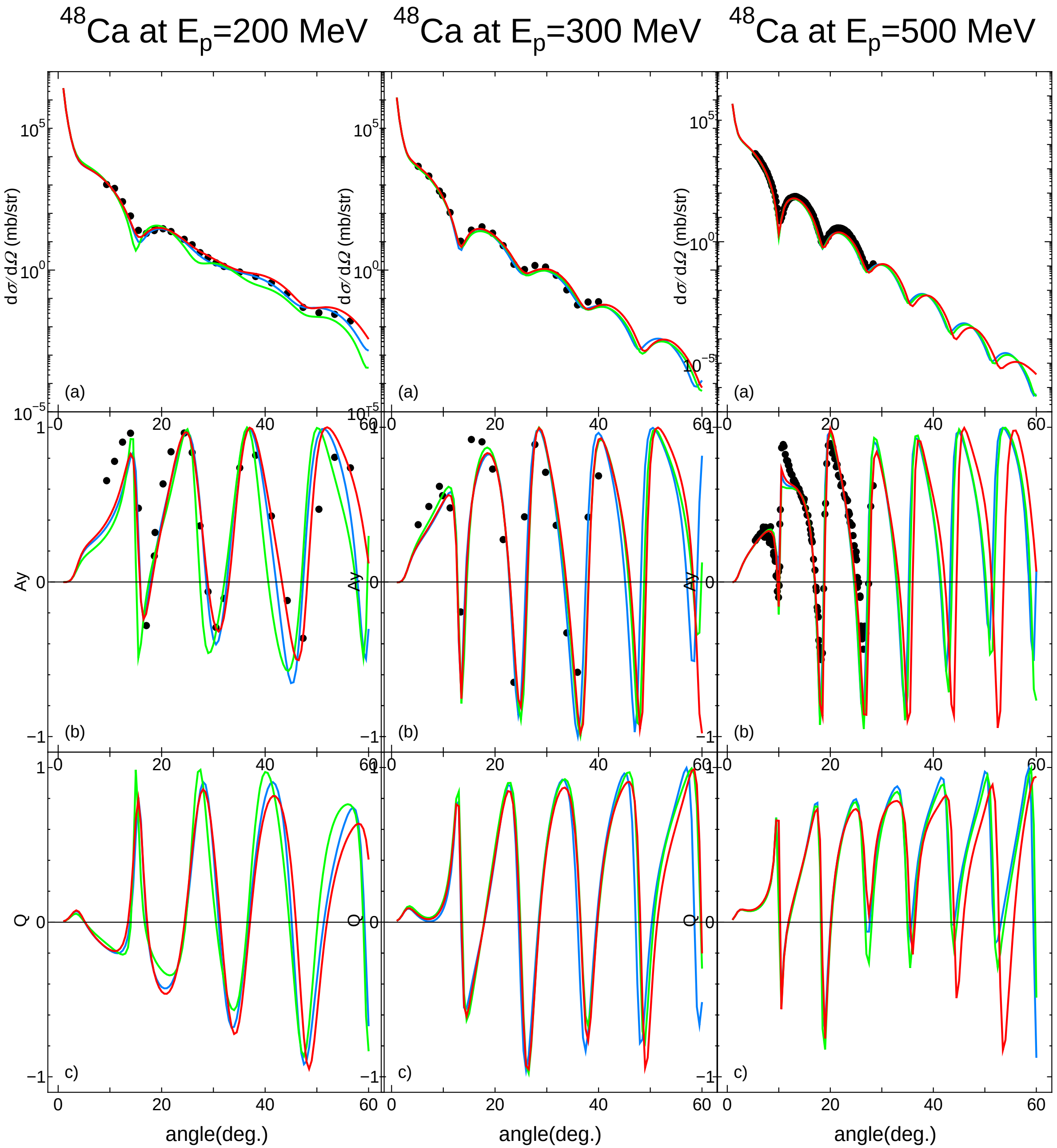
Relativistic Impulse Approximation

^{48}Ca

2nd

1st

med.



exp. data

A.E.Feldman et al.
G.W.Hoffman et al.

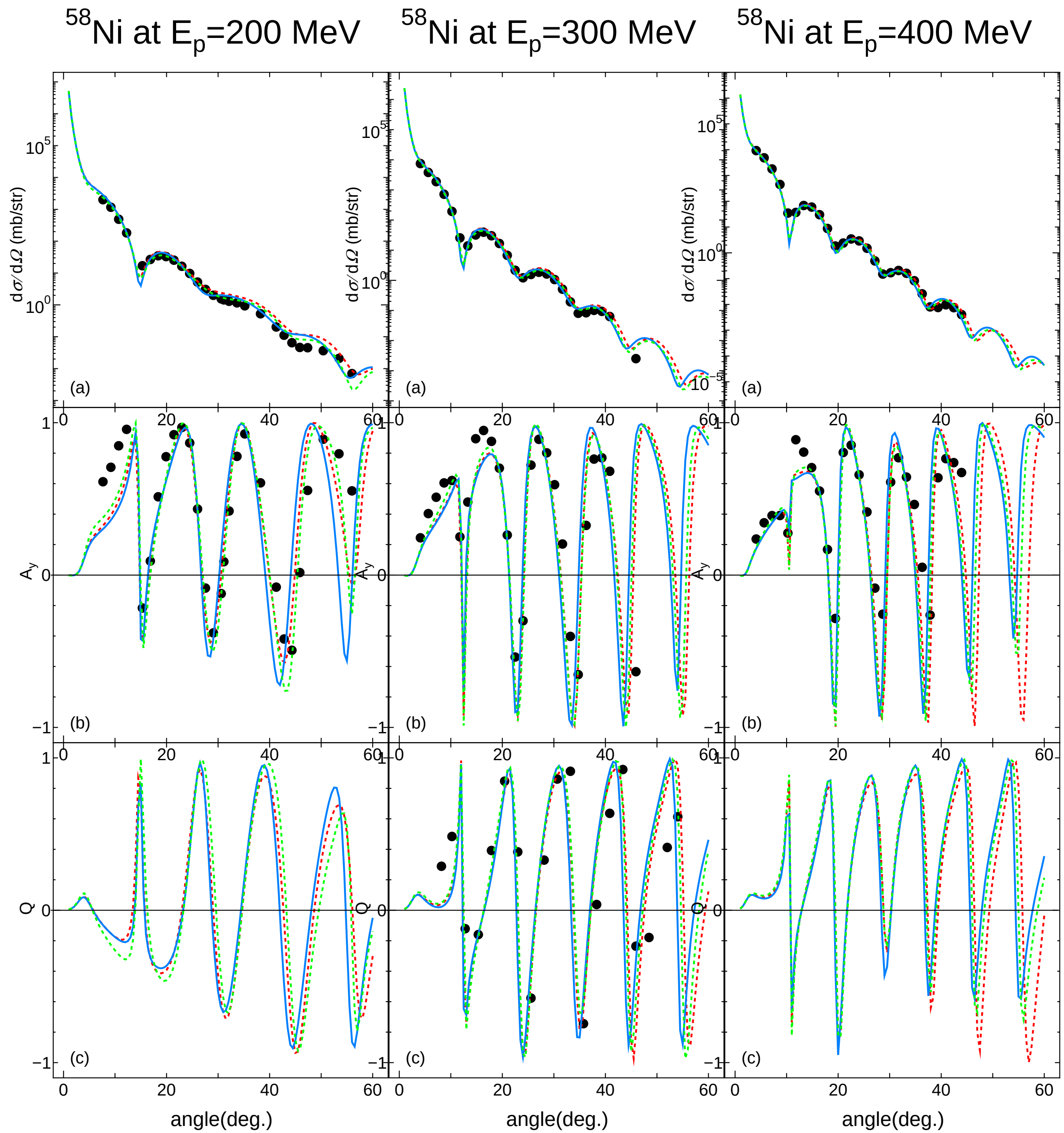
Relativistic Impulse Approximation

^{58}Ni

2nd

1st

med.



exp. data

H.Sakaguchi et al.
PRC57(1998)1749

Relativistic
Impulse
Approximation

- $^{60,68,74}Ca$ at 200 MeV
- $^{60,68,74}Ca$ at 300 MeV
- $^{60,68,74}Ca$ at 400 MeV
- $^{60,68,74}Ca$ at 500 MeV

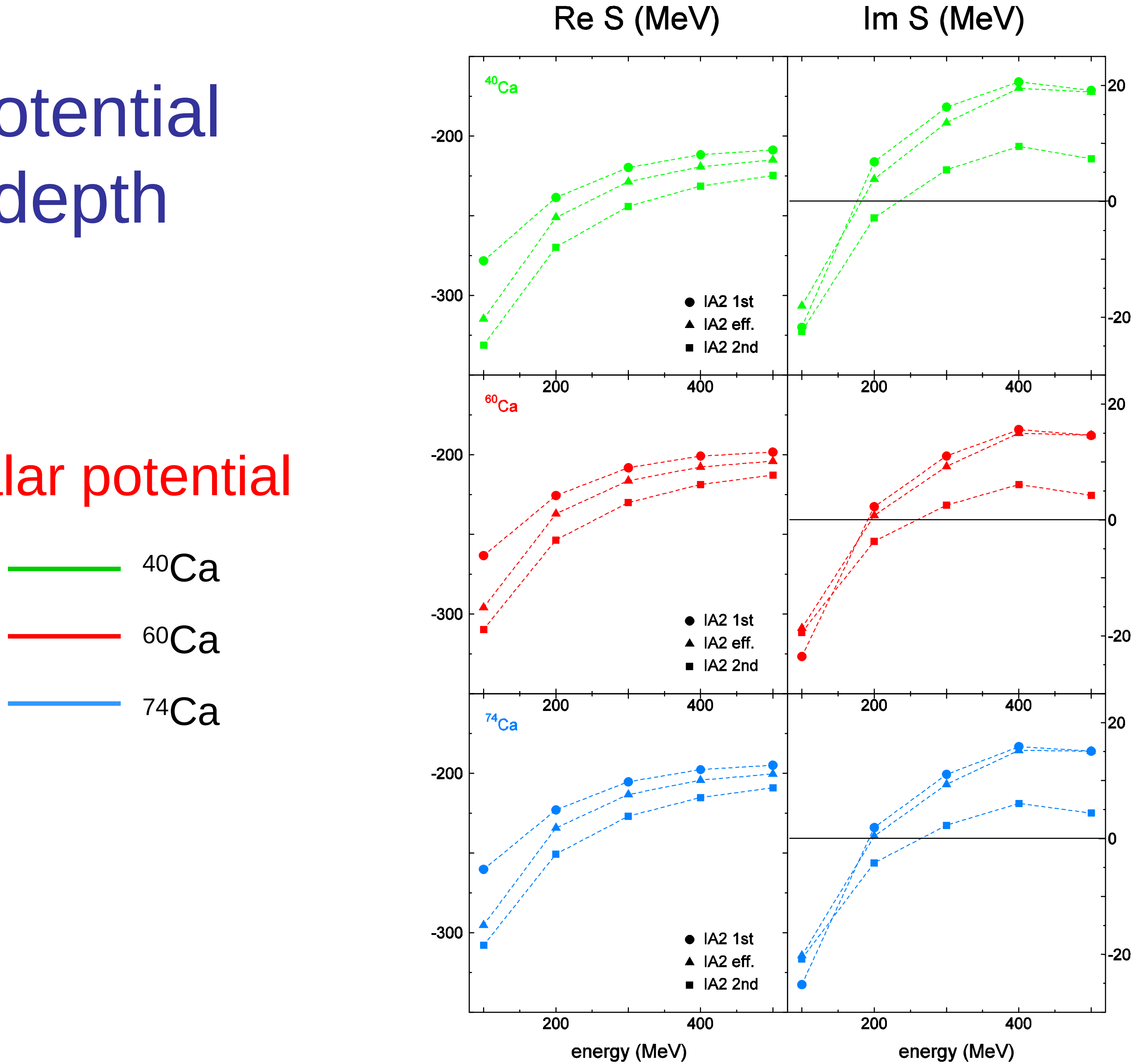
Relativistic
Impulse
Approximation

- $^{48-64}$Ni at 200 MeV
- $^{48-64}$Ni at 300 MeV
- $^{48-64}$Ni at 400 MeV
- $^{48-64}$Ni at 500 MeV

- $^{66-82}$Ni at 200 MeV
- $^{66-82}$Ni at 300 MeV
- $^{66-82}$Ni at 400 MeV
- $^{66-82}$Ni at 500 MeV

potential depth

scalar potential



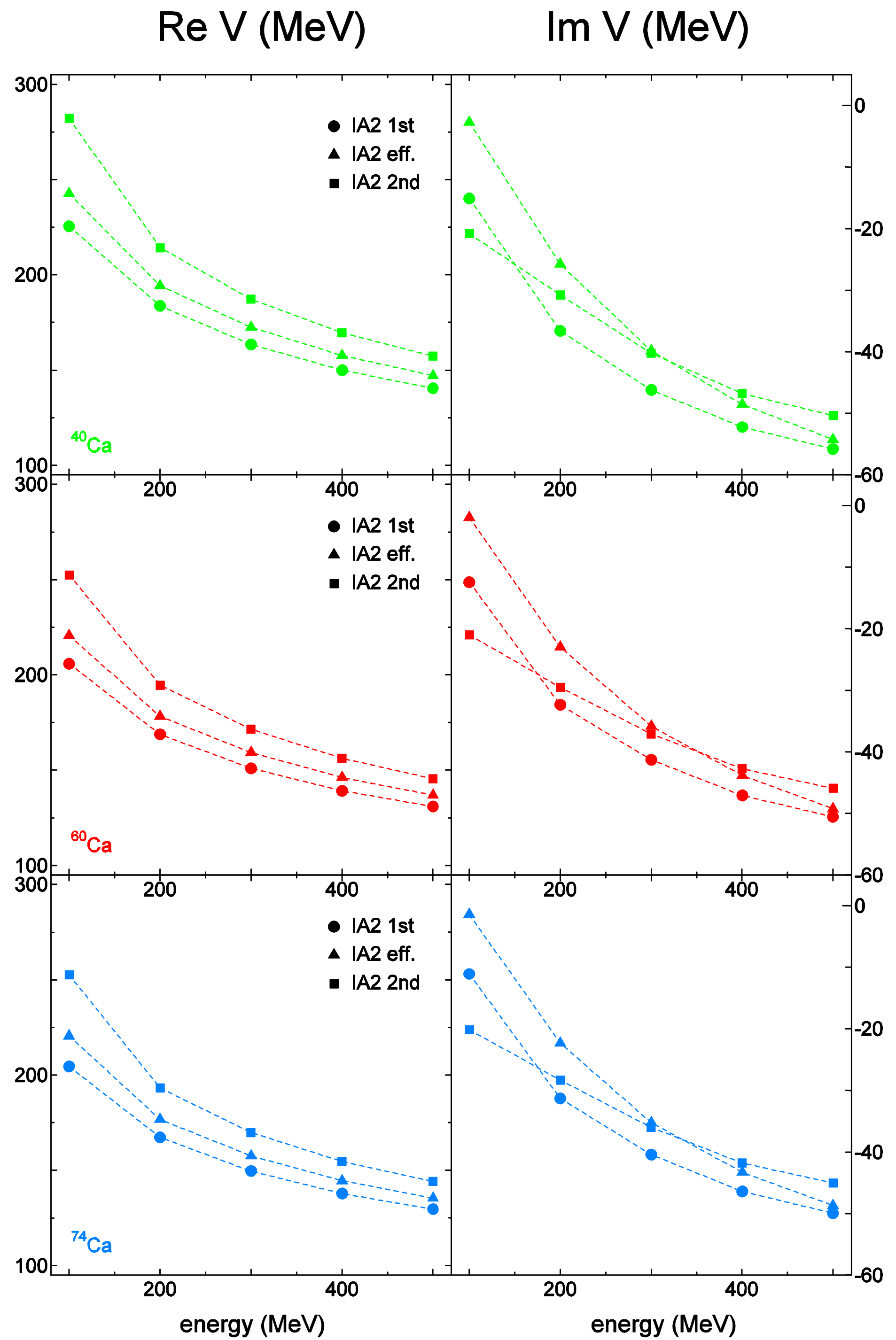
potential depth

vector potential

^{40}Ca

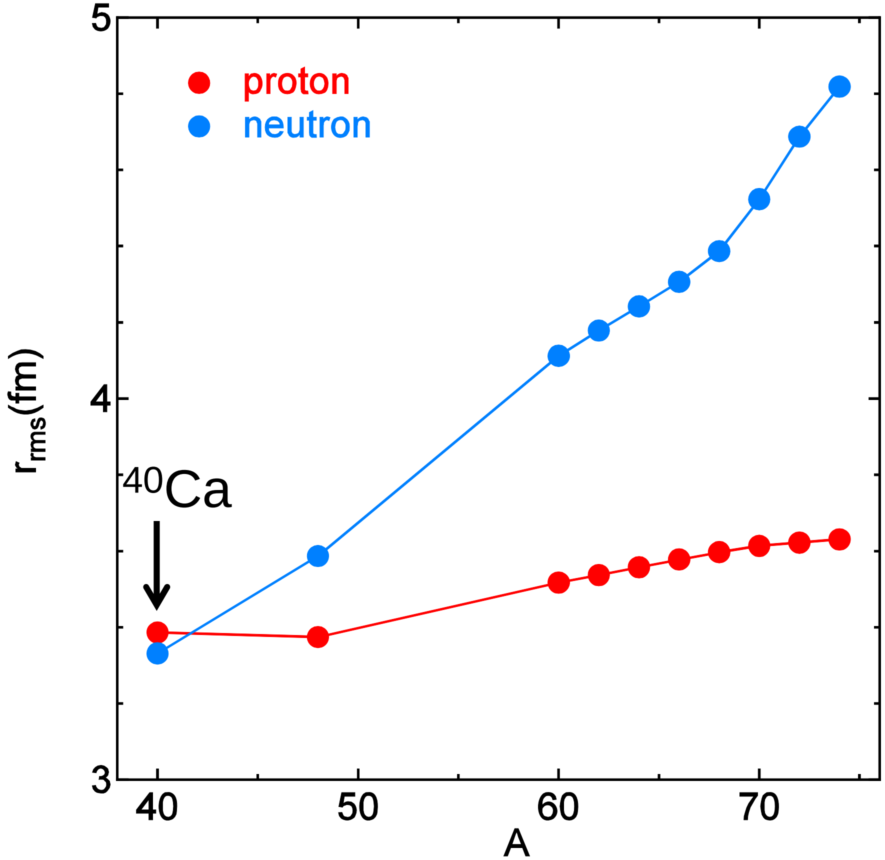
^{60}Ca

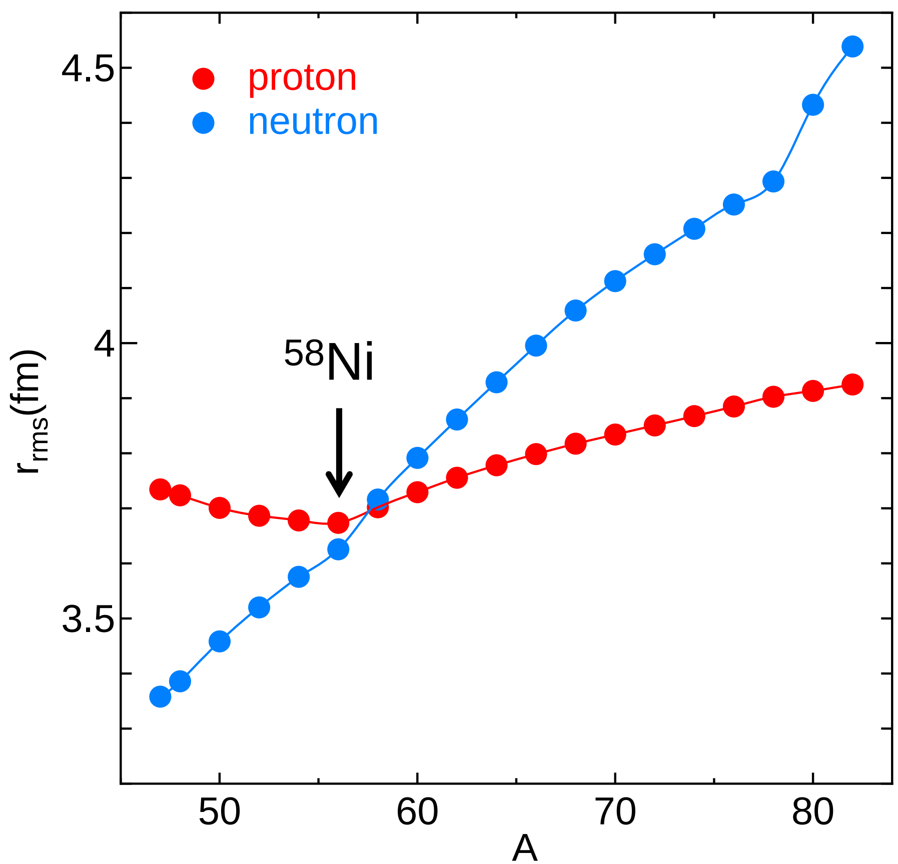
^{74}Ca



relation between A & Δ

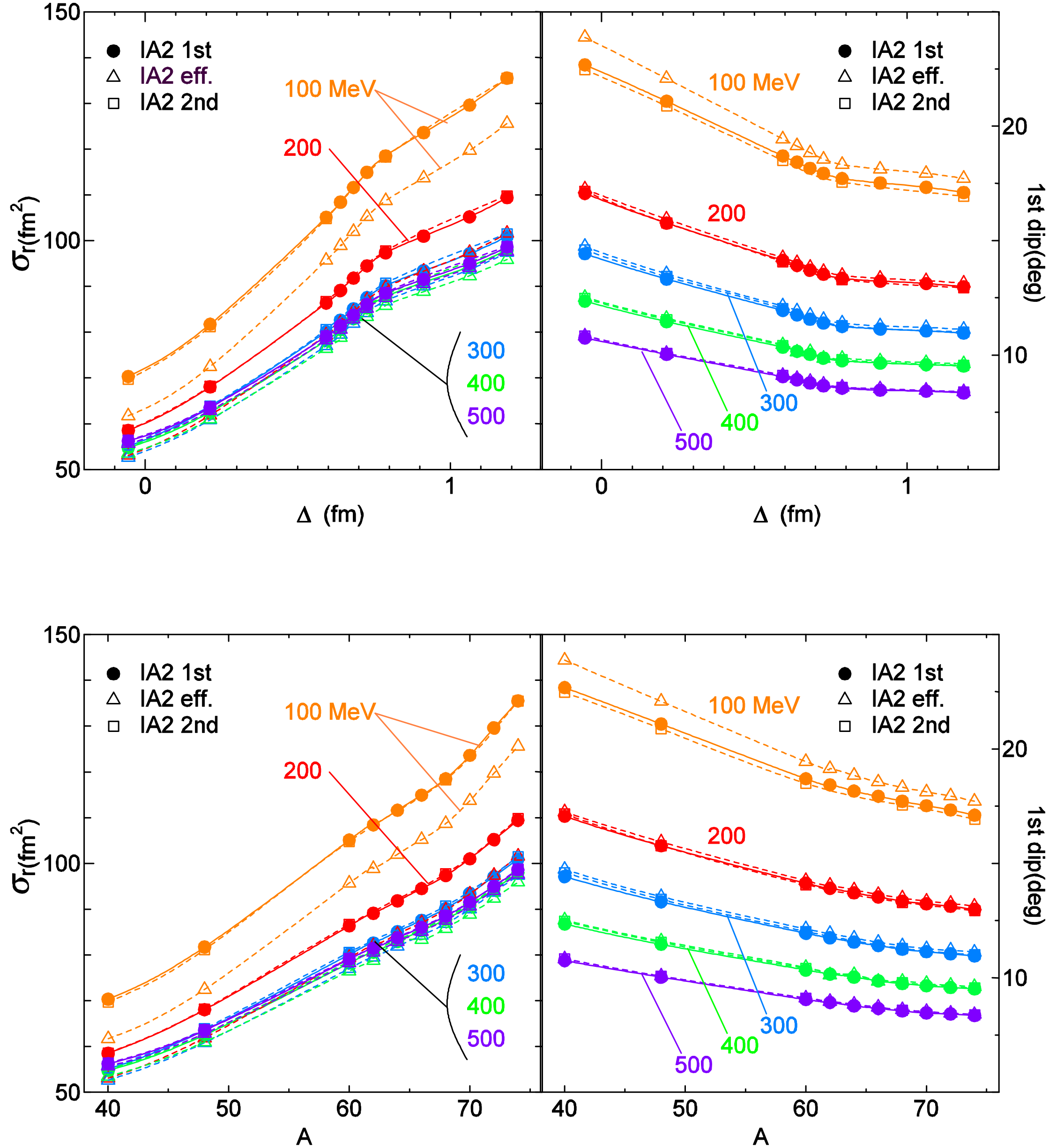
$$\Delta = \sqrt{<r_n{}^2>} - \sqrt{<r_p{}^2>}$$





reaction cross sections & 1st dip position

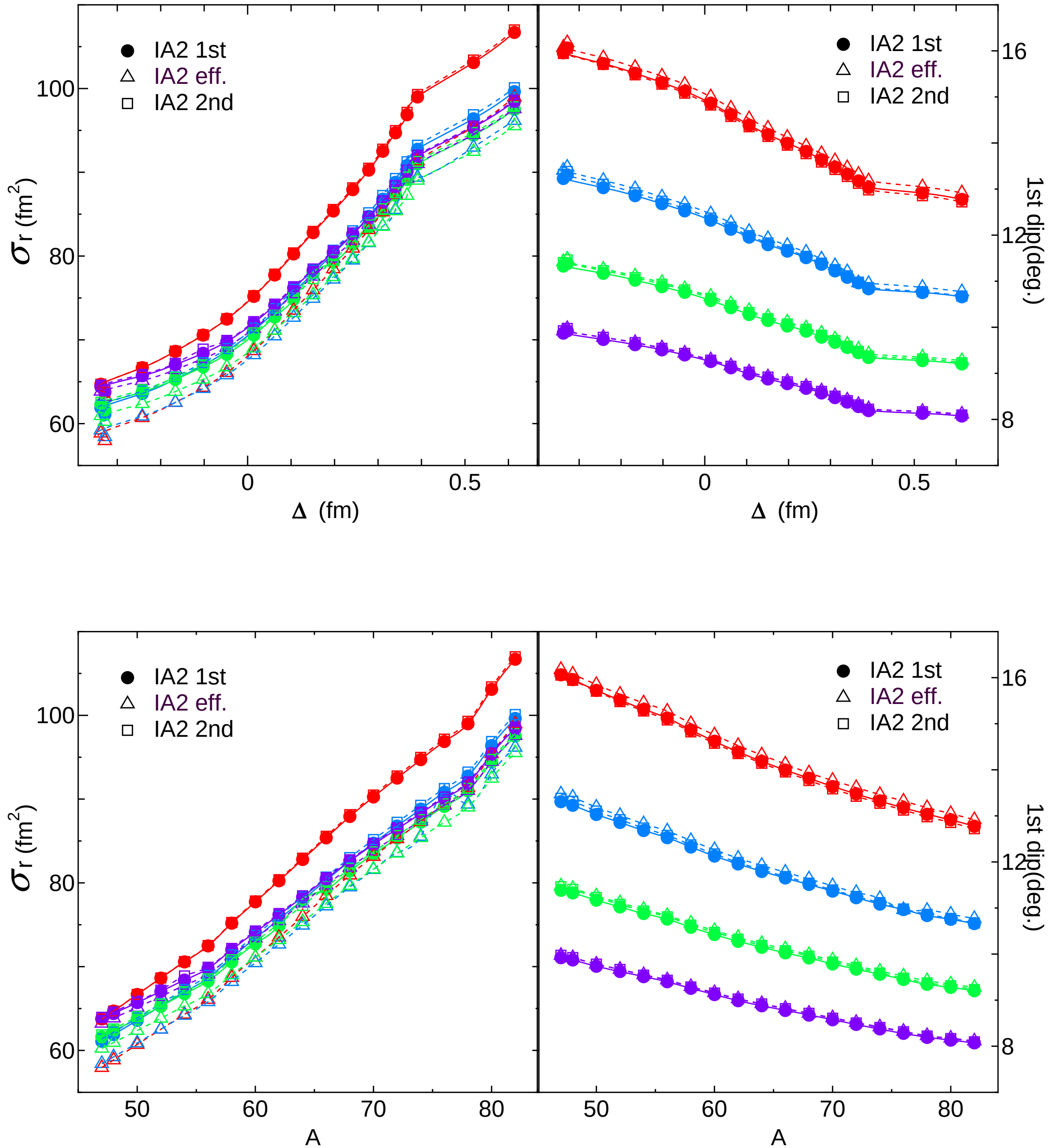
Ca isotopes



reaction cross sections & 1st dip position

Ni
isotopes

200 MeV
300
400
500



study for neutron distribution

1. K.Kaki & S.Hirenzaki, int.J.Mod.Phys. E, 2(1998) 167-178 ← ^{60}Ca
2. K.Kaki, int.J.Mod.Phsy.E, 13(2004) 787-799 ← ^{208}Pb

normalized by

$$A - Z = 4\pi \int \rho(r) r^2 dr$$

$$\rho(r) = \frac{\rho_0}{1 + \exp\{(r - r_0)/a\}}$$

radial parameter

diffuseness parameter

*proton distributions are fixed to the charge or rmf density

Fourier transformation

$$\rho(q) = 4\pi \int_0^\infty j_0(qr) \frac{\rho_0}{1 + e^{(r - r_0)/a}} r^2 \, dr$$

$$\approx \frac{4\pi\rho_0}{q^3} \frac{\pi q a}{\sinh(\pi q a)}$$

$$\times \left\{ \pi q a \cdot \coth(\pi q a) \sin(q r_0) - q r_0 \cdot \cos(q r_0) \right\}$$

differential cross section

the 1st Born approx.

t-ρ form

$$\frac{d\sigma}{d\Omega} = \left| f_B(\theta) \right|^2 = \left(\frac{\mu}{2\pi\hbar} \right)^2 \left| t(q) \right|^2 \rho^2(q)$$

1st dip position ⟺ $\rho(q) = 0$

mean-square radius

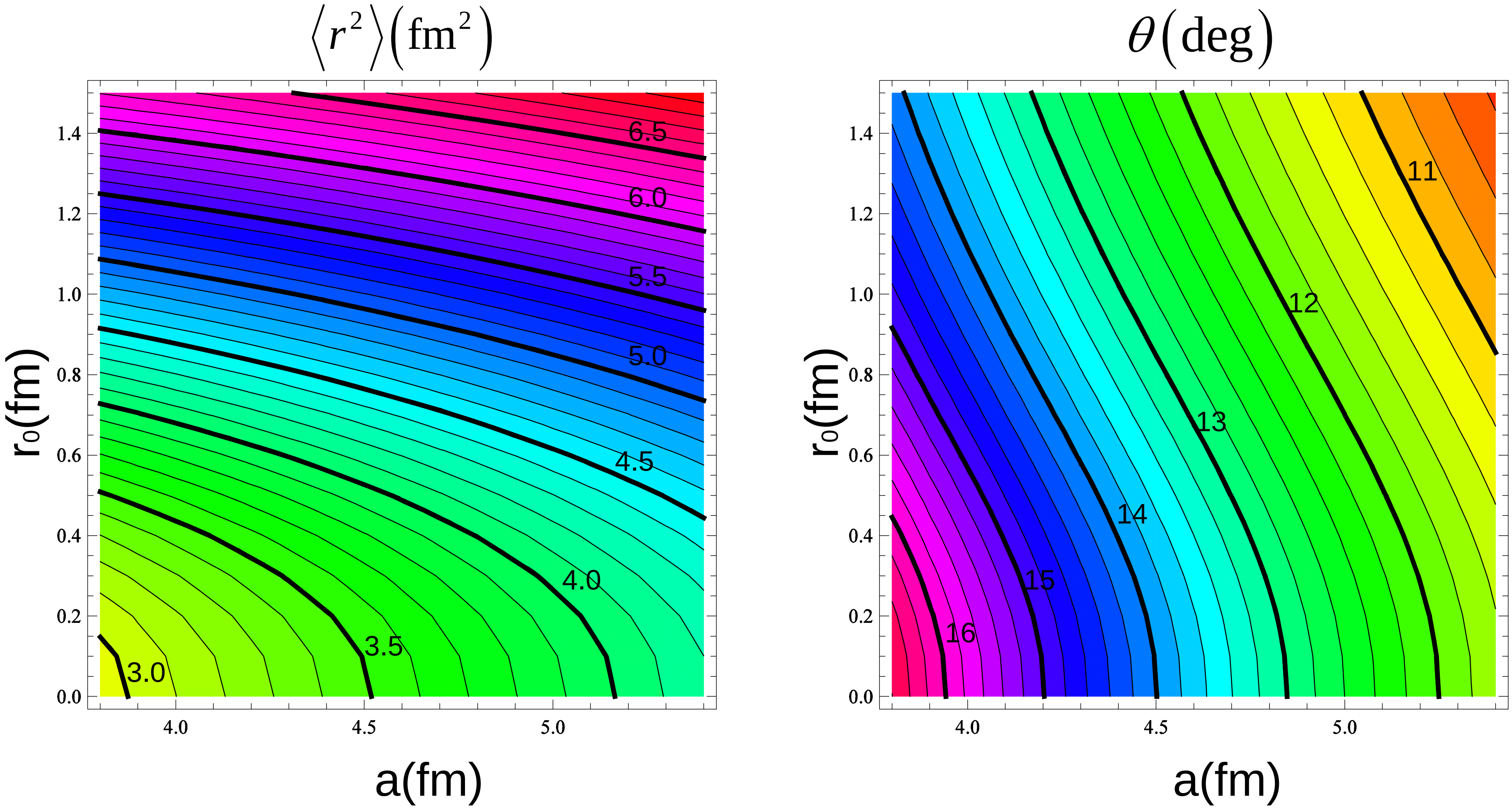
$$\left\langle r^2 \right\rangle = 4\pi \int r^2 \rho(r)\, r^2 dr \Big/ 4\pi \int \rho(r)\, r^2 dr$$

$$= \frac{1}{5}\left[7(\pi a)^2 + 3r_0^2 \right]$$

analytic function of the parameters

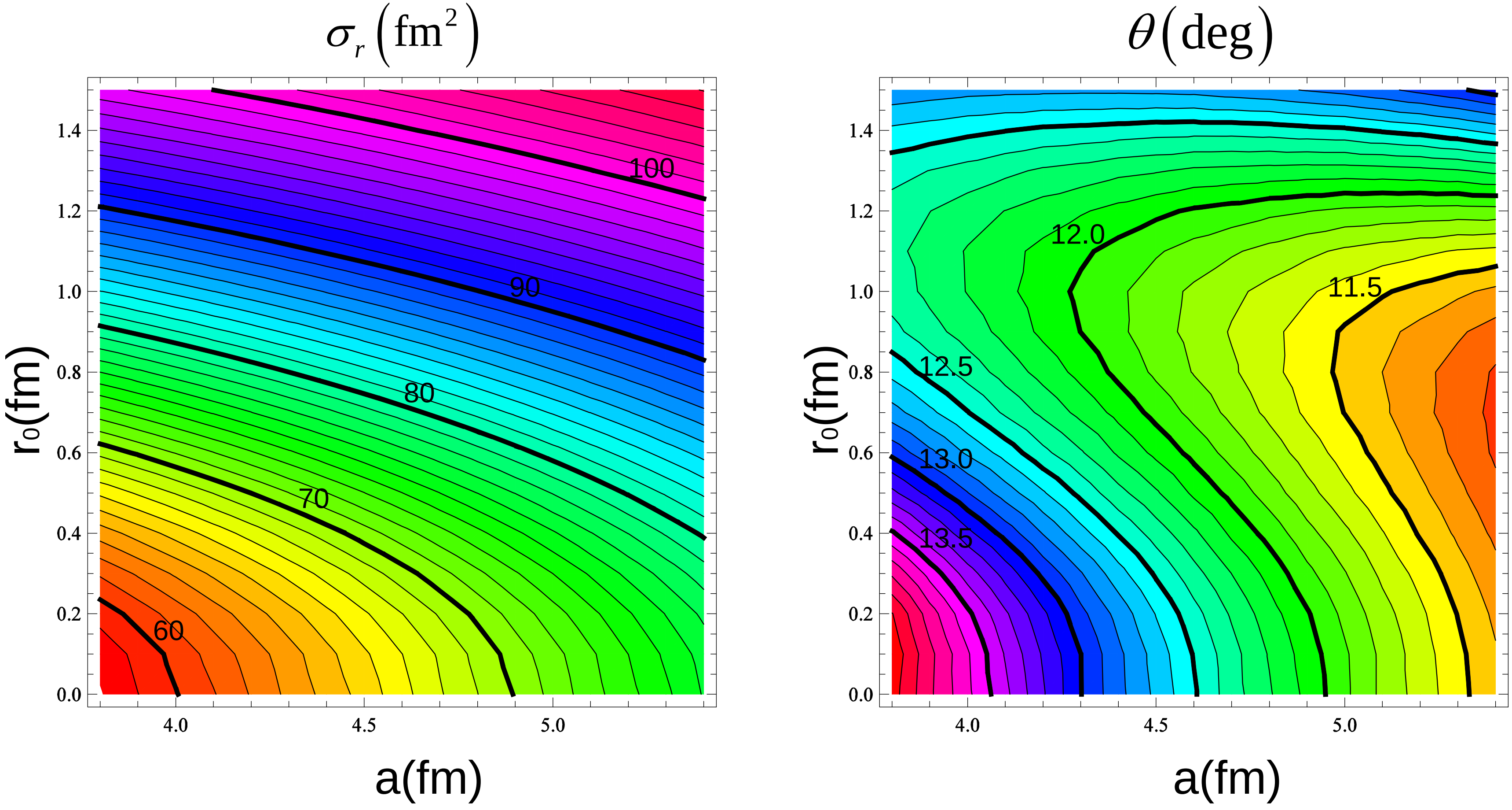
contour map of msr & dip with respect to r_0 & a

analytic cal.

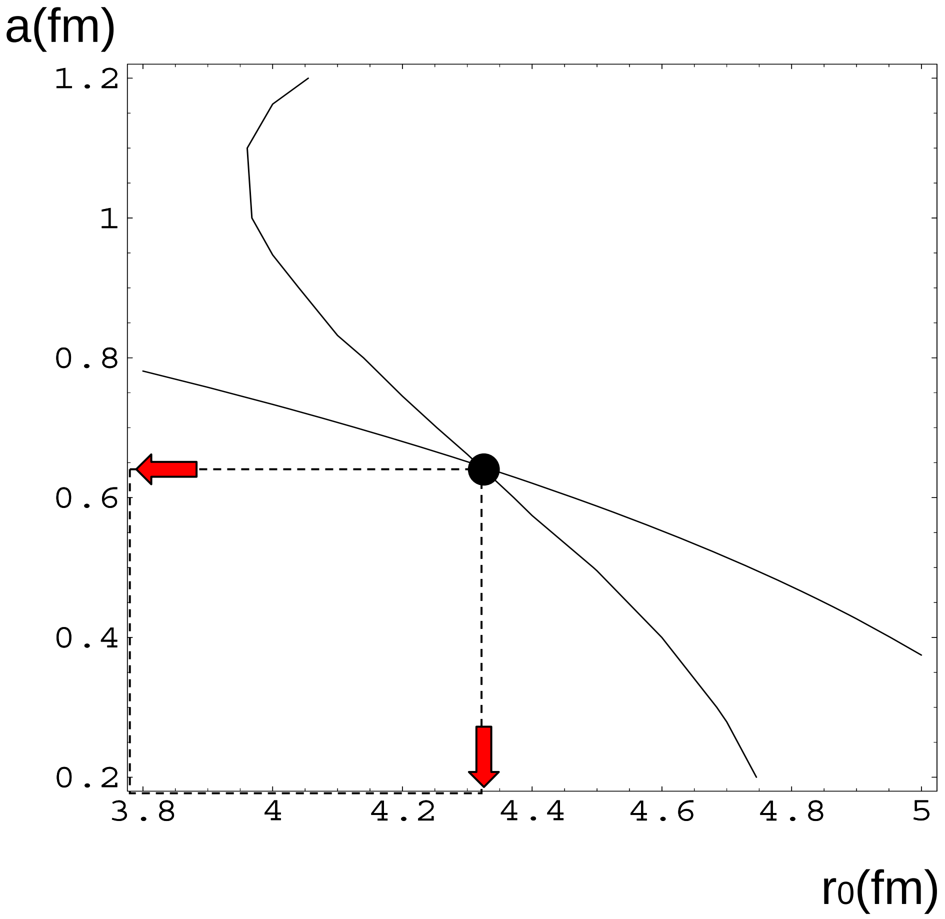


contour map of rcs & dip with respect to r_0 & a

observevables



to determine parameters



^{60}Ca rmft

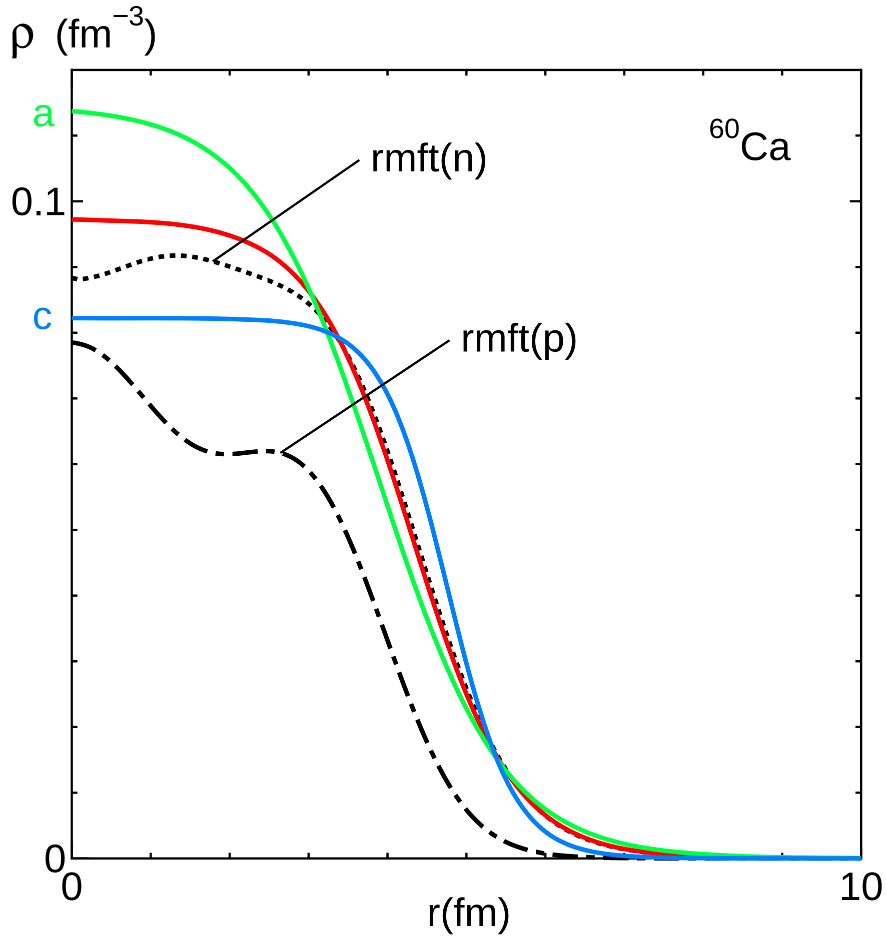
$$\sigma_r = 75.34 \ \text{(fm}^2\text{)}$$

$$\theta = 12.22 \ \text{(deg.)}$$

$$r_0 = 4.32 \ \text{(fm)}$$

$$a = 0.64 \ \text{(fm)}$$

obtained density distribution for neutron



summary

- observables of proton-elastic scattering from $^{40,48,60\text{-}74}$Ca nuclei and $^{48\text{-}82}$Ni nuclei
    - incident energies : 200, 300,400 & 500 MeV
    - Relativistic Impulse Approximation ⟹ IA2 parameters
    - Relativistic Mean Field Theory ⟹ nuclear densities
- medium effects
    - reaction cross section ⟹ a little bit smaller
- multiple scattering effect (2nd order potential)
    - contributions ⟹ at rather low energy & larger angle
    - reaction cross section ⟹ a little bit larger
    - dip positions ⟹ slightly different but not significant

conclusion

- to determine the neutron distribution of unstable nuclei
  - RIA with IA2 parameter
  - k=0 for incident proton energies : 200-500 MeV
  - medium & multiple scattering effects : not significant role

    both in reaction cross section and dip positions

near future

- target nucleus ; Ni isotopes, Sn isotopes
  - energy range : 200-500 MeV
  - 1^{st} order calculations of RIA with OF
  - contour maps for the WS parameters: r_0 , a

Relativistic Impulse Approximation

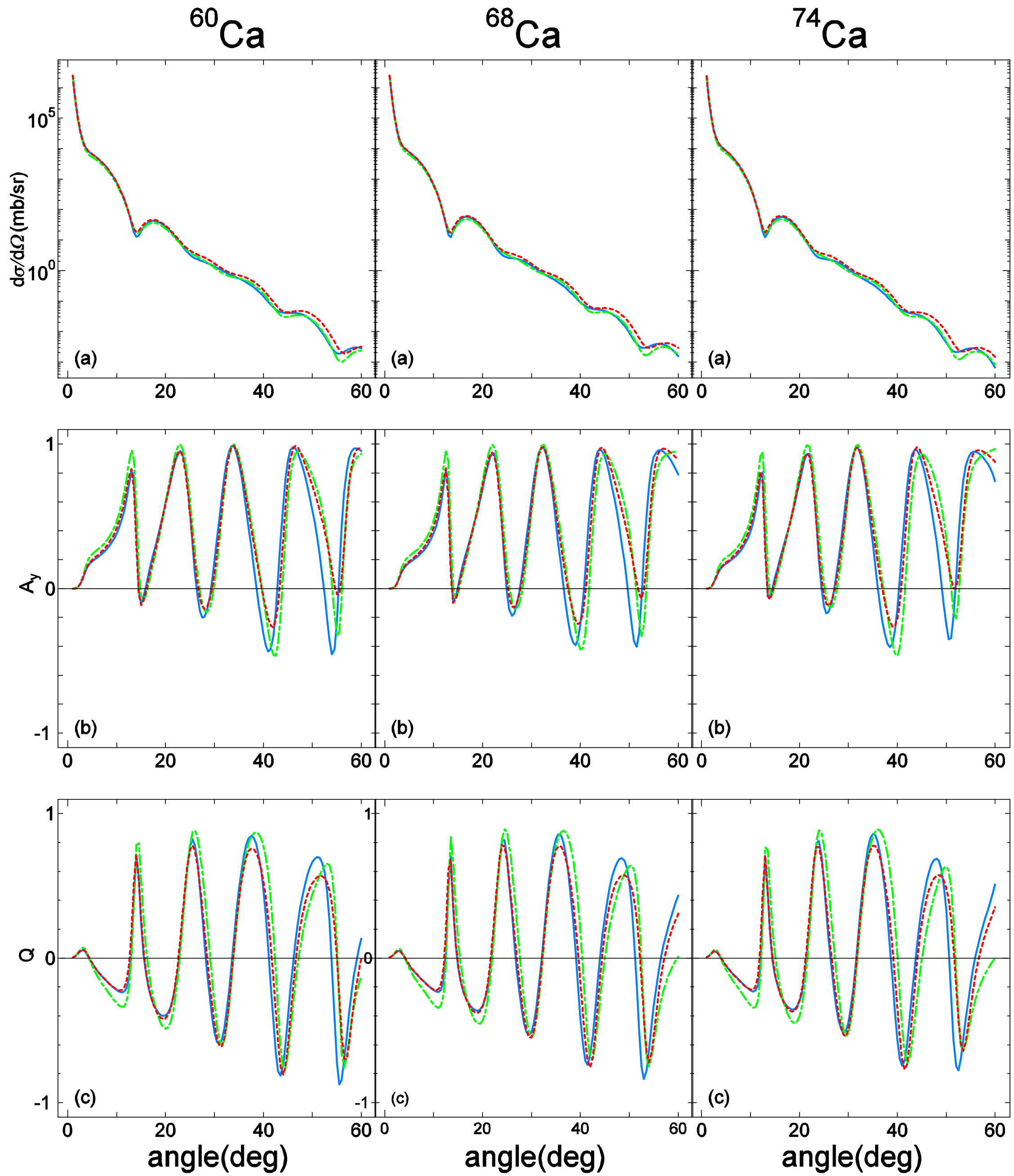
$^{60,68,74}Ca$

200 MeV

2nd

1st

med.



E_p =200 MeV

Relativistic Impulse Approximation

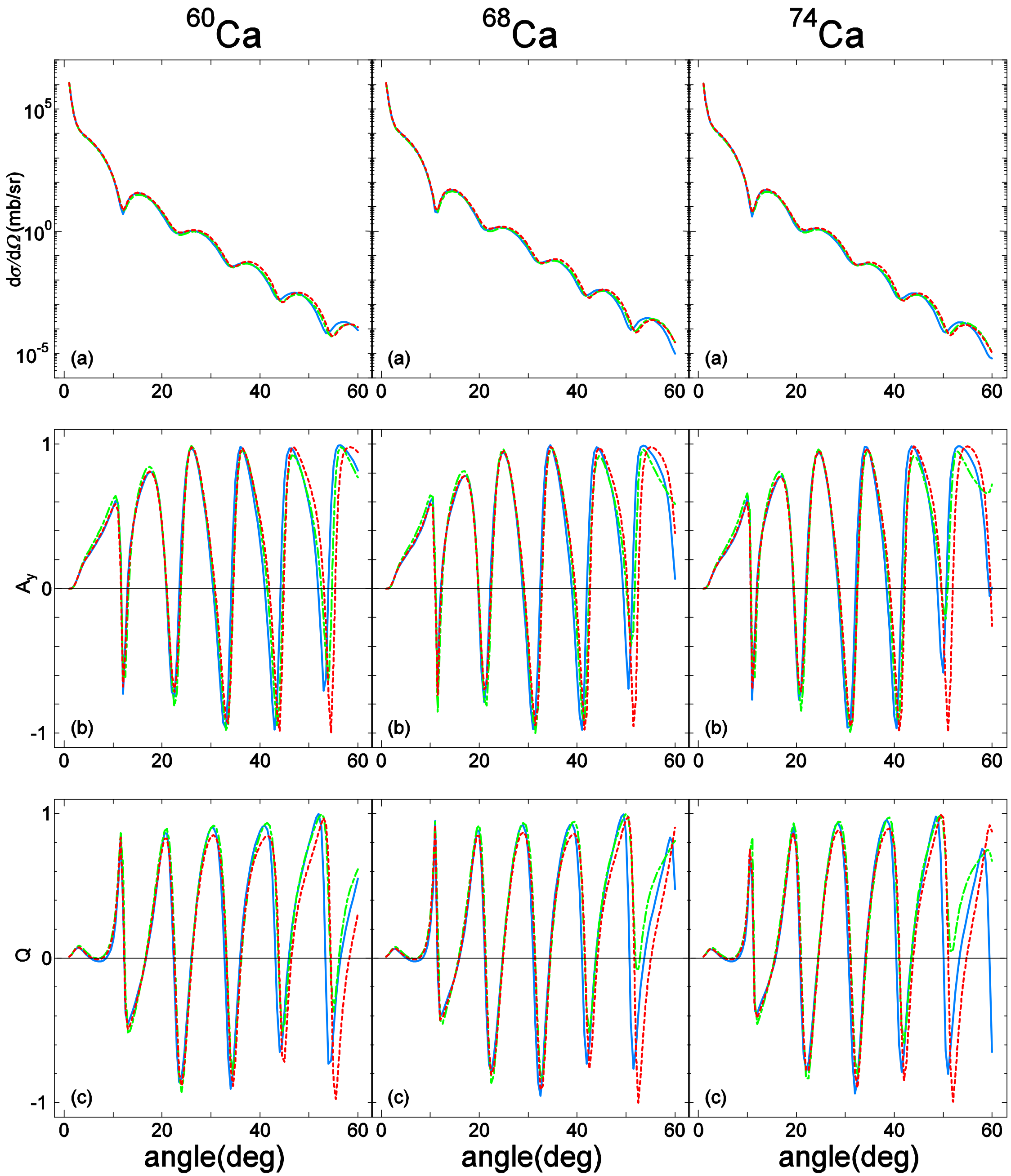
^{60,68,74}Ca

300 MeV

2nd

1st

med.



E_p =300 MeV

Relativistic Impulse Approximation

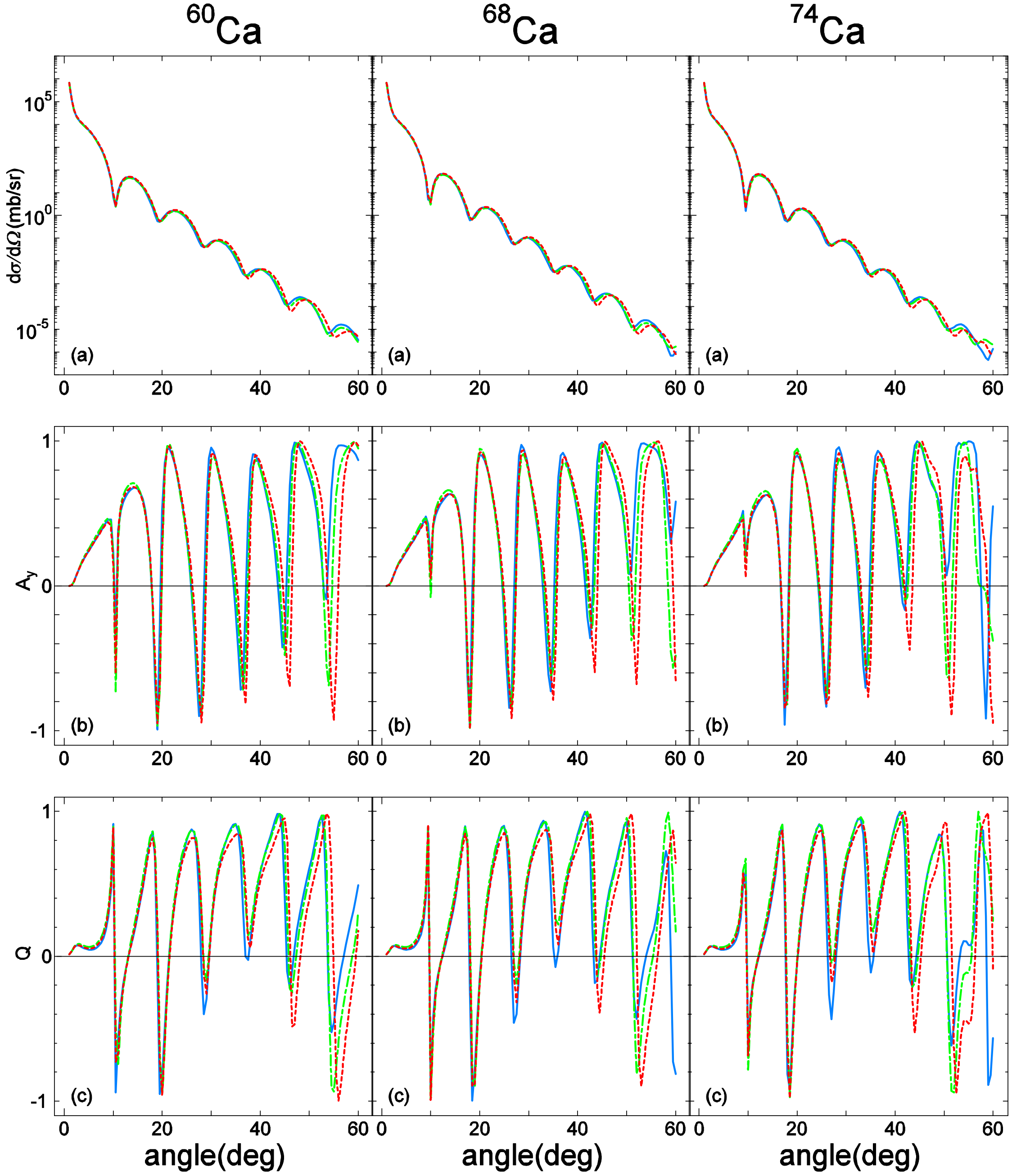
$^{60,68,74}Ca$

400 MeV

- - - - 2nd

—— 1st

-·-·- med.



E_p =400 MeV

Relativistic Impulse Approximation

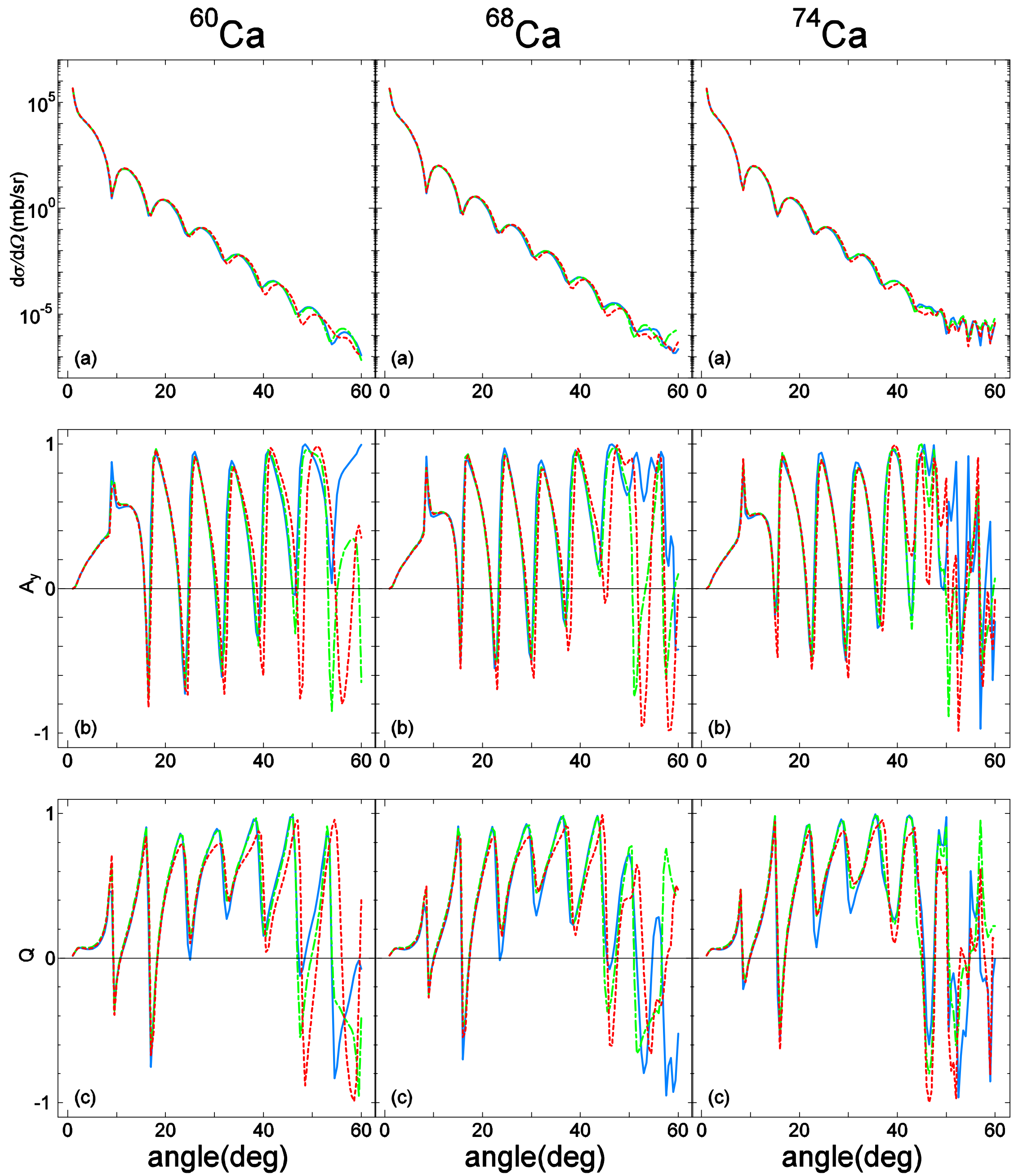
^{60,68,74}Ca

500 MeV

- - - - 2nd

—— 1st

-·-·- med.



E_p =500 MeV

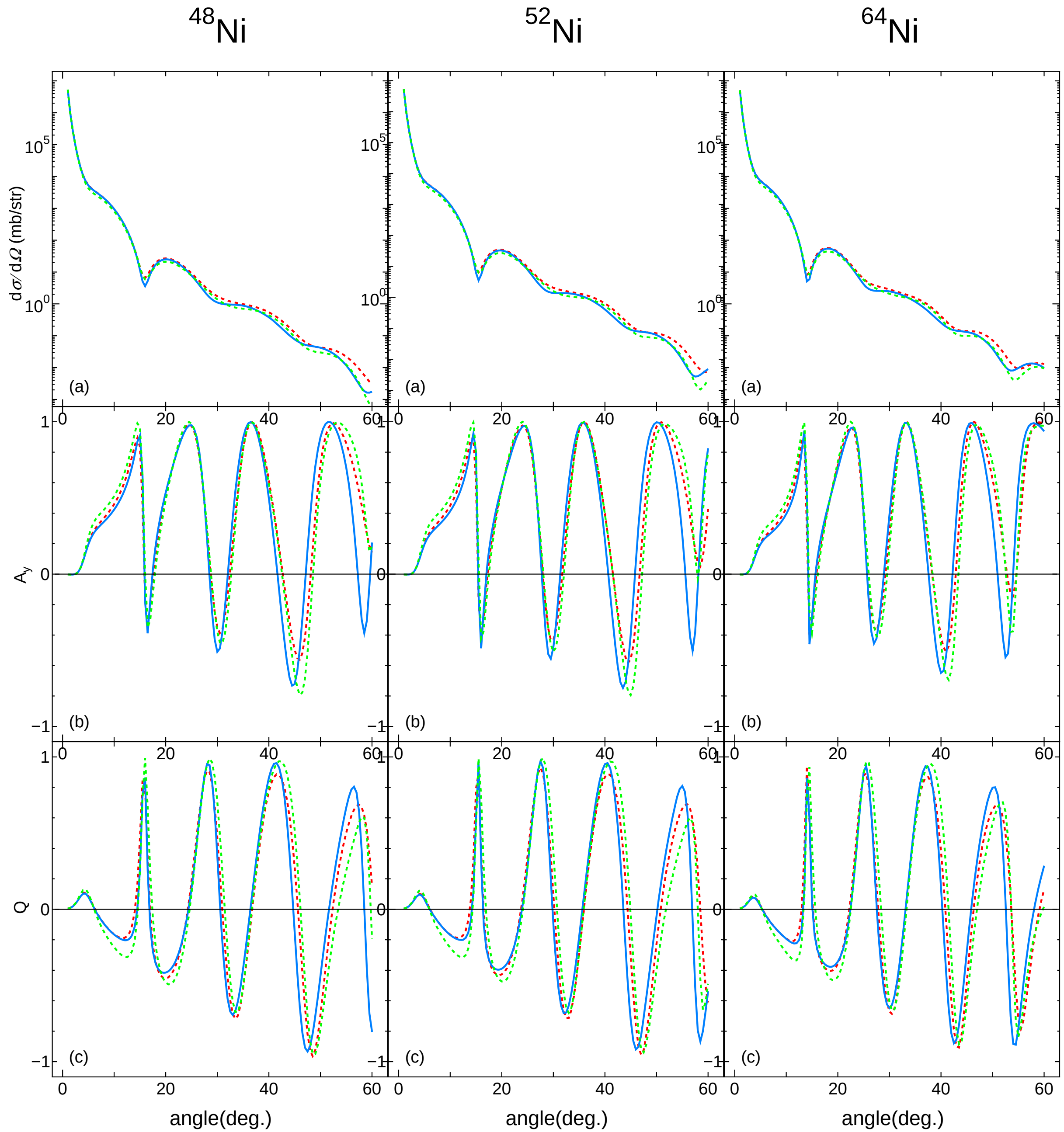
Relativistic Impulse Approximation

$^{48\text{-}64}$Ni

- - - - 2nd

—— 1st

-·-·- med.



E_p =200 MeV

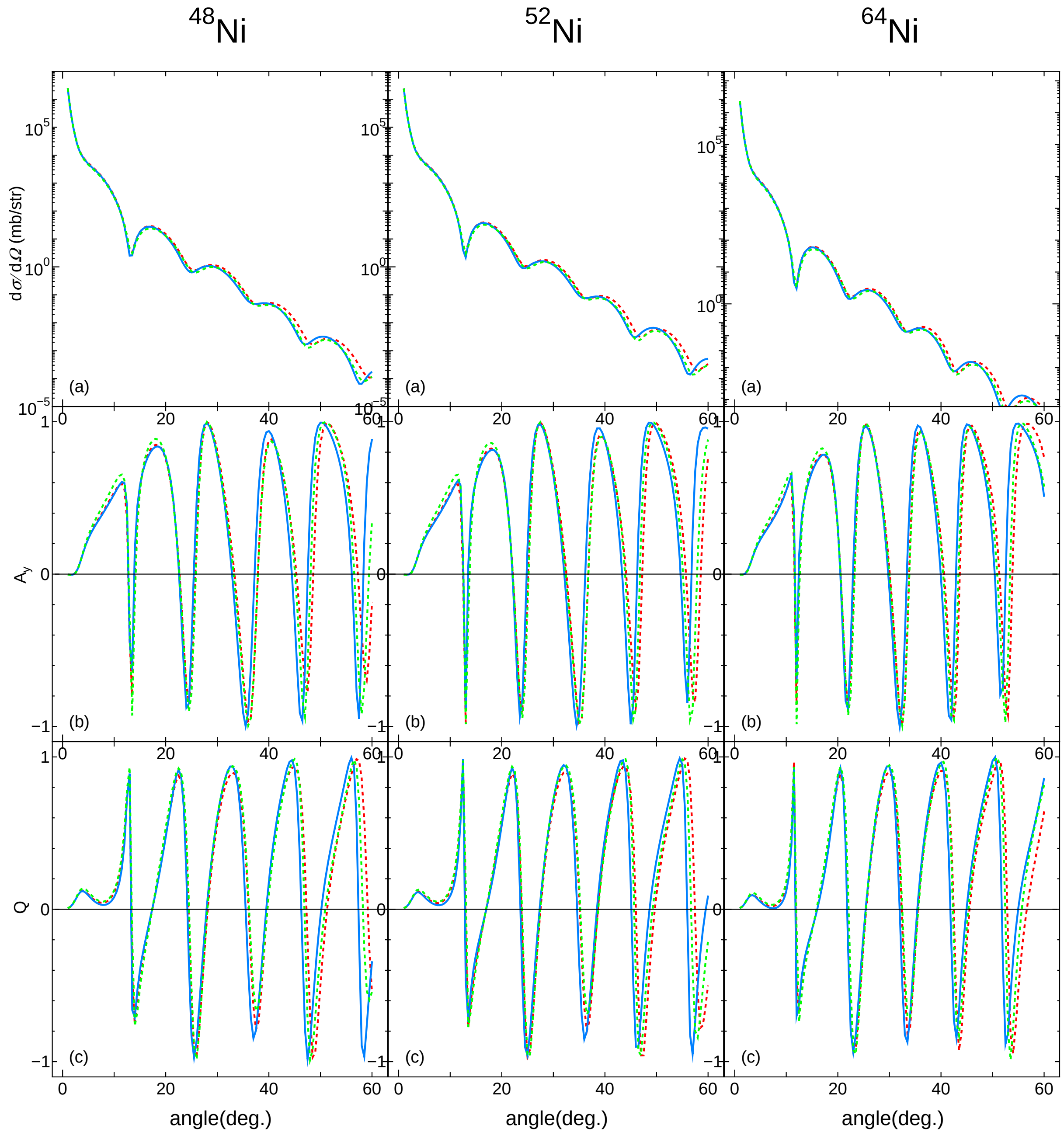
Relativistic Impulse Approximation

$^{48\text{-}64}$Ni

- - - - 2nd

—— 1st

-·-·- med.



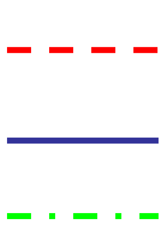
E_p =300 MeV

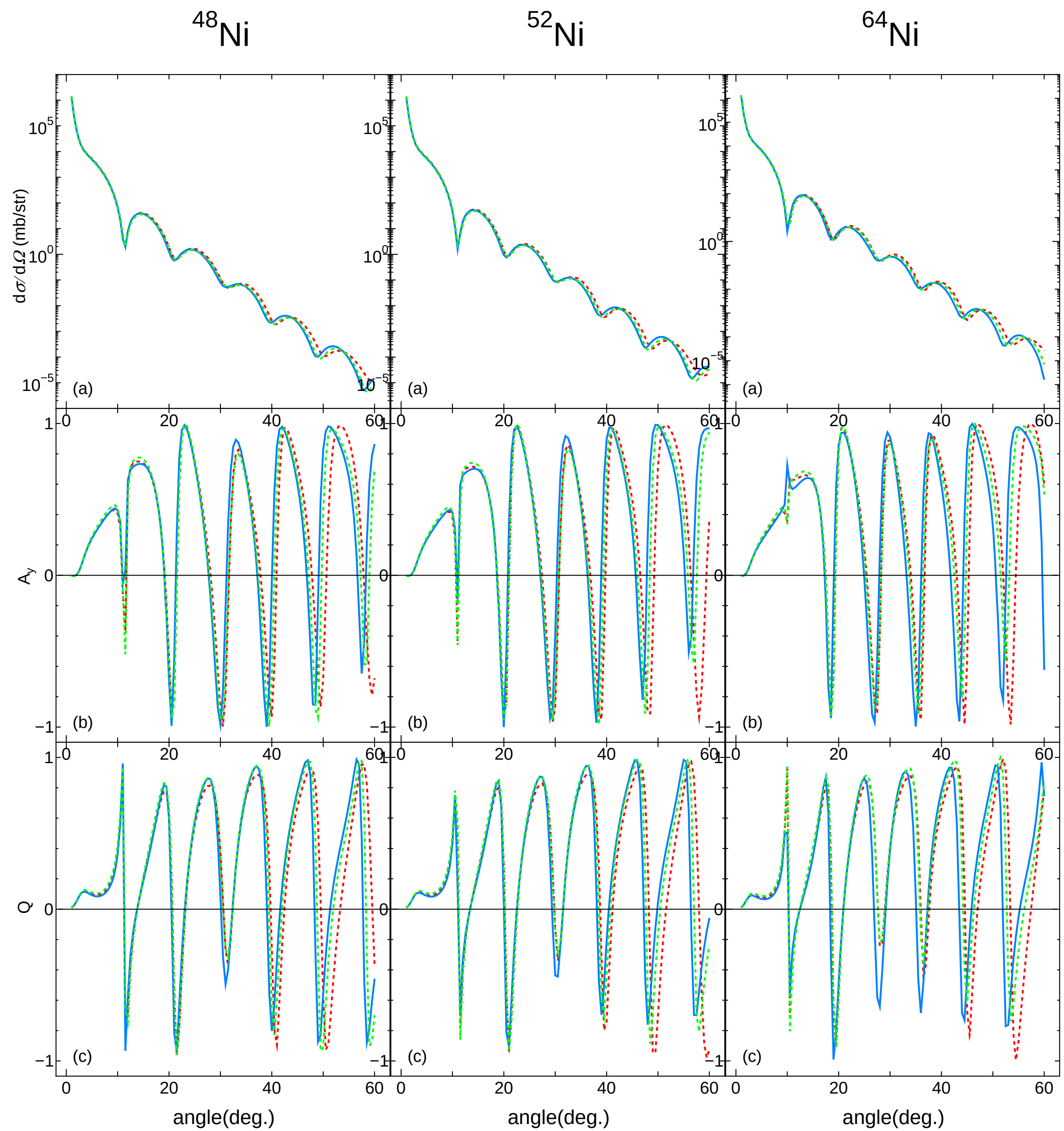
Relativistic

Impulse

Approximation

$^{48\text{-}64}$Ni

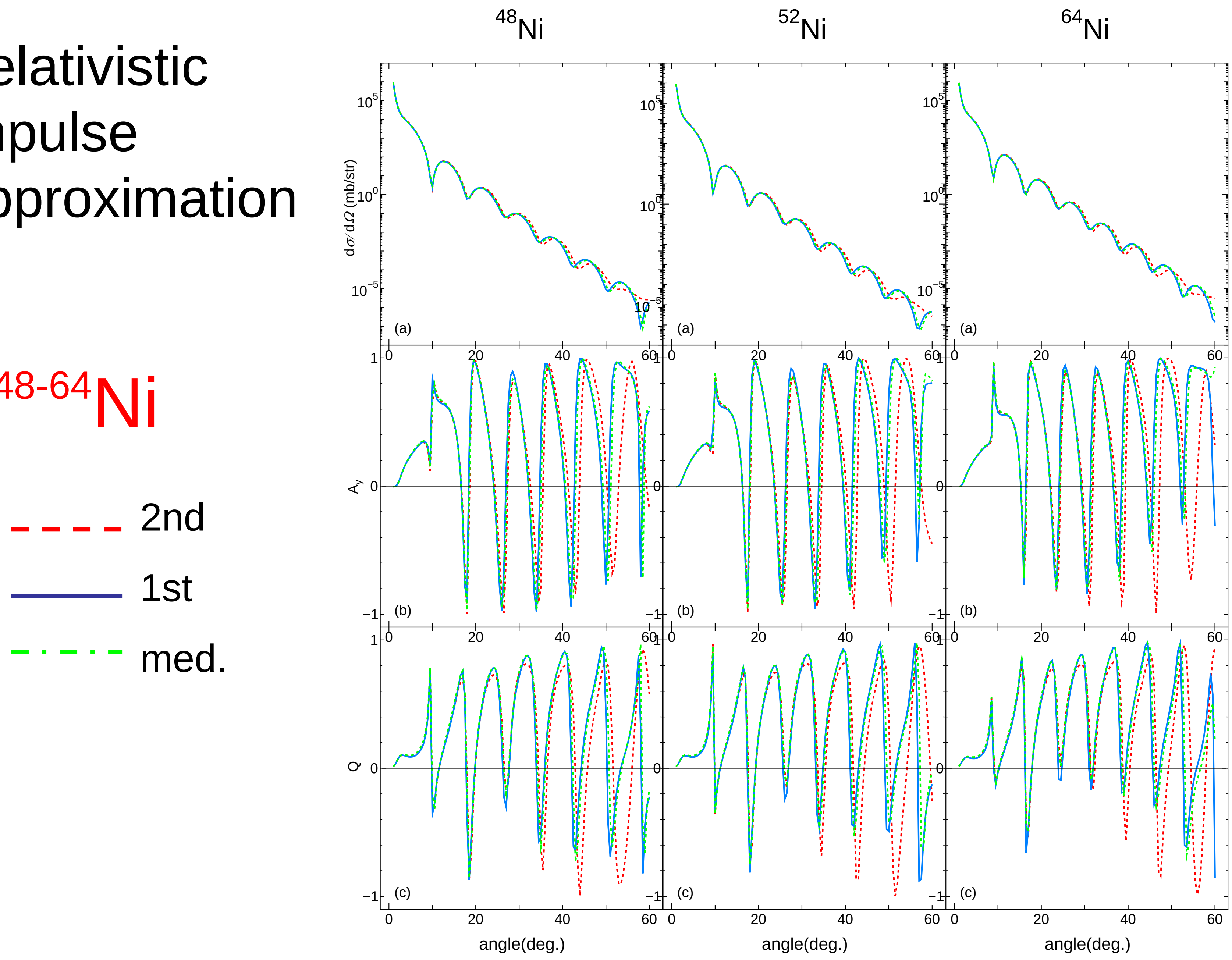




E_p =400 MeV

Relativistic Impulse Approximation

$^{48-64}$Ni



E_p =500 MeV

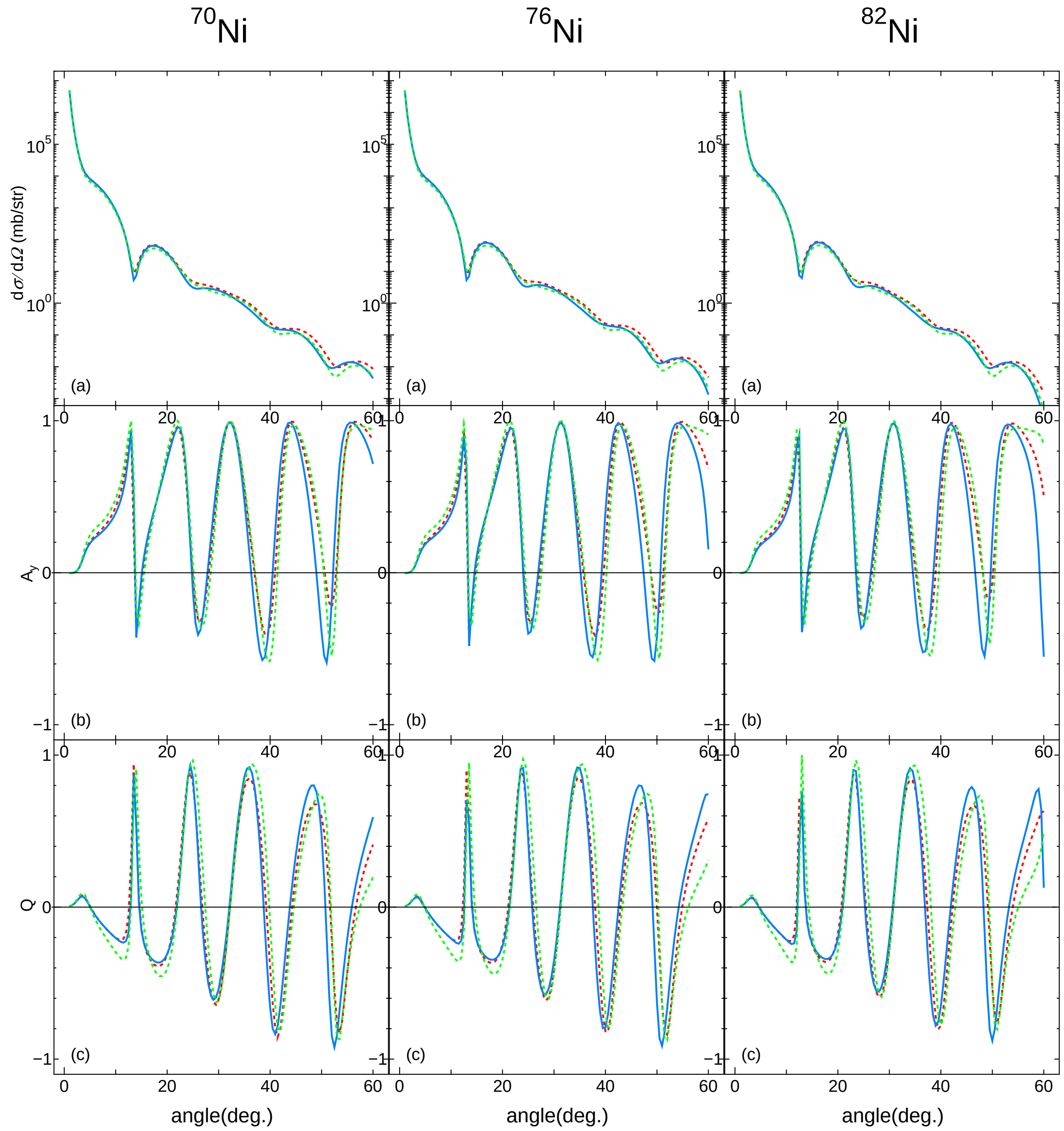
Relativistic Impulse Approximation

$^{70-82}Ni$

- - - - 2nd

—— 1st

-·-·- med.



E_p =200 MeV

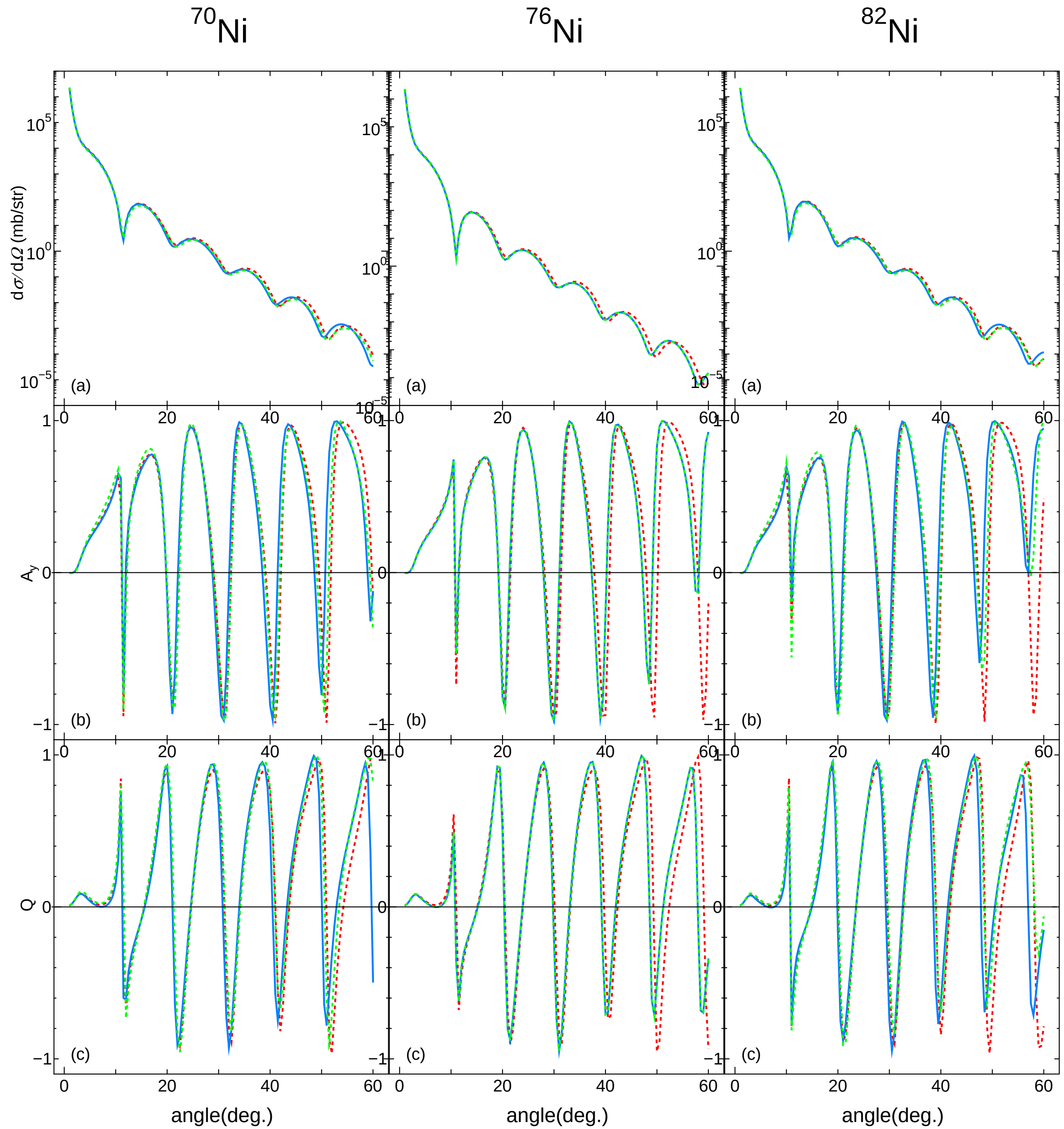
Relativistic Impulse Approximation

$^{70\text{-}82}Ni$

- - - - 2nd

—— 1st

- · - · med.



E_p =300 MeV

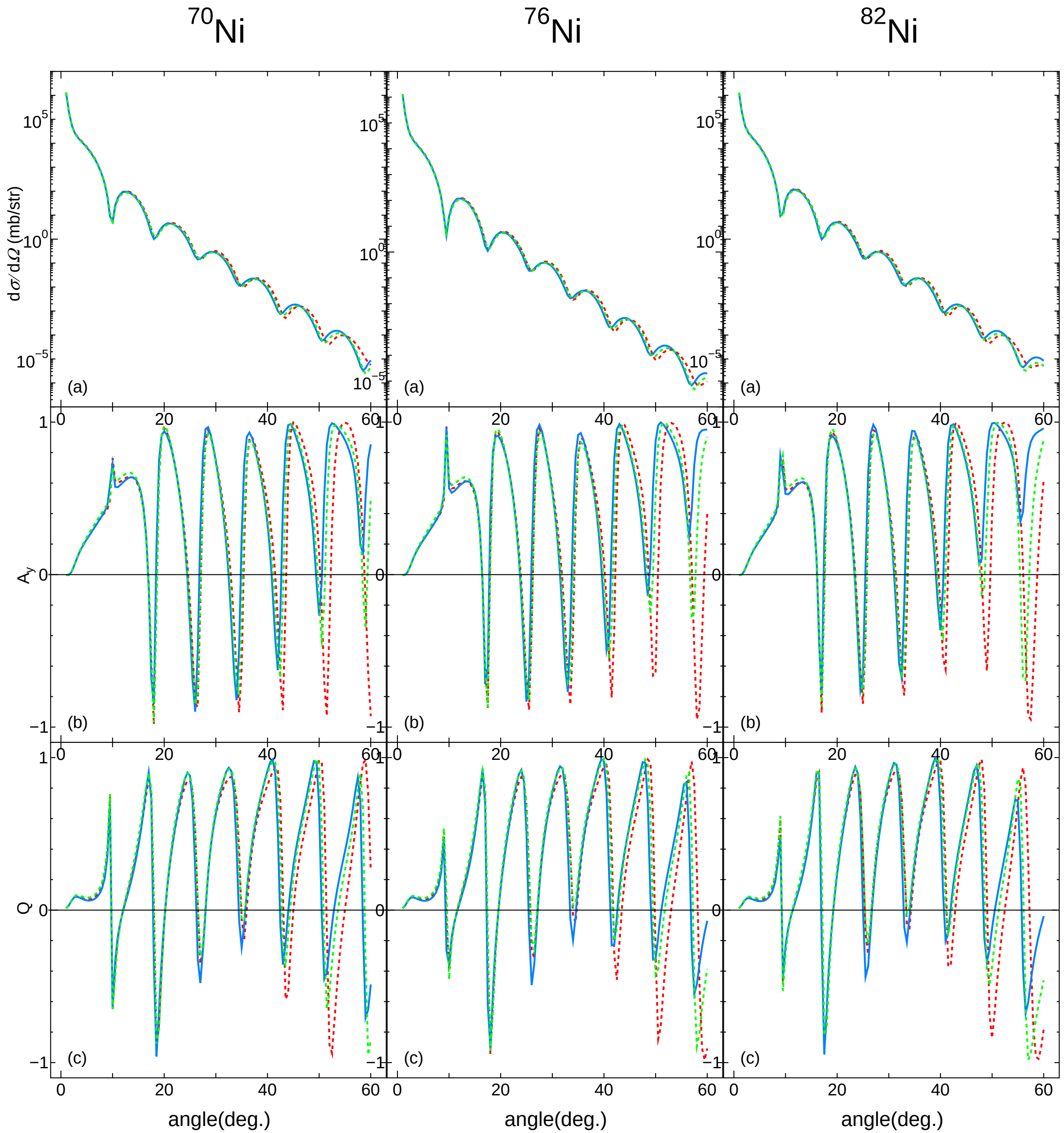
Relativistic Impulse Approximation

$^{70-82}$Ni

- - - - 2nd

—— 1st

- · - · med.



E_p =400 MeV

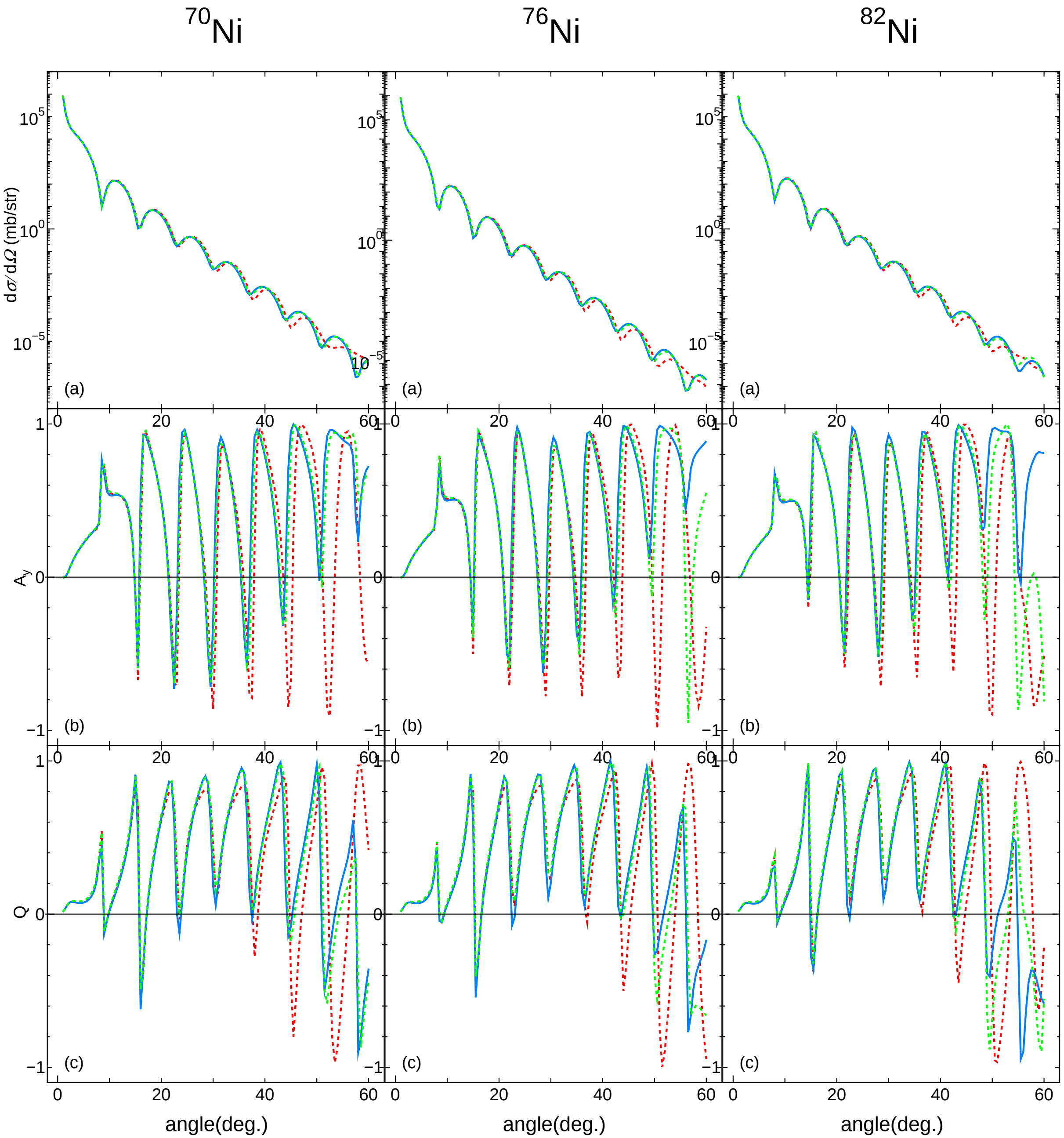
Relativistic Impulse Approximation

$^{70\text{-}82}Ni$

2nd

1st

med.



E_p =500 MeV