A Nonlinear Complementary Filter for Attitude Estimation with Dynamics Compensation of MARG Sensor

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***Abstract*— A novel complementary filter for three-dimensional (3D) attitude estimation is proposed. In order to deals with the nonlinearity of 3D attitude, the conventional complementary filter integrated the measured angular velocity corrected by the vector measurement, but it ignored the sensor dynamics which causes the lag. Authors' previous work compensated the sensor dynamics based on the idea that it can be divided into the linear transfer function and the nonlinear coordinate transformation. The properness and stability of the total transfer function are guaranteed by the designed filter inserted between the linear and nonlinear part. However, the disturbance strongly affects the accuracy of the azimuth angle due to the coordinate transformation. This paper aims to improve the estimation accuracy by combining the idea of correcting the measured angular velocity by the vector measurement and the sensor dynamics compensation only with the proper and stable transfer function. The characteristics of the proposed method are the following two points: 1) the relaxation of the complementary condition in order to make the non-proper and unstable function proper and stable, and 2) the filtering of the estimated vector by the sensor dynamics instead of the filtering of the measured vector by the inverse dynamics in order to avoid the magnification of the high frequency noise. Through the experiment for the fast and irregular motion under 5[Hz], the validity of the proposed method is verified.**

I. INTRODUCTION

Fast and irregular attitude variation of a mobile robot with short control period, such as a humanoid robot, is the target to be estimated for a high-rate feedback control. From the requirement that the measurement period is the same as the control rate, inertial sensors and magnetometer are available as attitude sensors. However, it is difficult to estimate the attitude accurately by the homogeneous sensor. The rate gyroscope, which is one of inertial sensors, can measure the attitude variation. However, its output includes the bias, so that the integral of the output suffers from the error accumulation. Another one of inertial sensors is the inclinometer. It measures the inclination from the direction of the gravity, but its dynamics causes the lag. The magnetometer can estimate the azimuth angle based on the comparison between its current output and the geomagnetism or its initial output. However, its measurement is disturbed by the magnetic field of the surrounding environment. Therefore, the combination of the heterogeneous sensors such as MARG (Magnetic, Angular Rate, and Gravity) sensor is required.

Many attitude estimation techniques using MARG sensor were proposed. Those are roughly divided into two types: the Kalman filtering approach [1], [2], [3] and the complementary filtering approach[4], [5], [6], [7], [8], [9]. Although those filters are the same as the result[10], designs of them are different. The Kalman filter[11] is designed in the time domain based on the noise property of each signal. However, its design is difficult. On the other hand, the complementary filter is designed in the frequency domain based on the frequency property of each signal which is empirically known, so that it is easier to design it than Kalman filter.

In order to apply the complementary filtering approach to the three-dimensional (3D) attitude estimation, it is necessary to take the nonlinearity of the attitude into consideration. For the nonlinearity, some methods which integrate the measured angular velocity corrected by the vector measurement were proposed [5], [6], [7], [8]. Madgwick et al. [7] designed a filter based on the gradient direction of the error between the measured and estimated vector. Mahony et al. [8] formulated three types of nonlinear complementary filters which use the attitude matrix as the attitude representation and discussed the stability of them. Those filters assume that the sensor dynamics is negligible, but the dynamics of each sensor are not always complementary. Thus, in the target frequency domain to be measured, there are some frequency domains where the accuracy is degraded. In contrast, some attitude estimators with the sensor dynamics compensation were proposed [12], [9]. In the authors' previous work [9], the nonlinear dynamics of each sensor is divided into the linear transfer function and the nonlinear coordinate transformation, and each property is compensated. The inverse transfer function tends to be non-proper and unstable, so that the designed filter is inserted between the linear and nonlinear part in order to make the total transfer function for each sensor proper and stable. However, the azimuth angle computed by the coordinate transformation is strongly affected by the disturbance included in the magnetometer's output.

The goal of this paper is to estimate 3D attitude accurately in the target frequency domain. For this purpose, a complementary filter which fuses the idea of correcting the measured angular velocity by the vector measurement and

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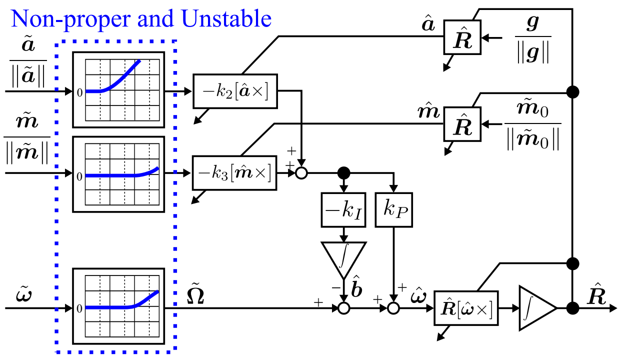
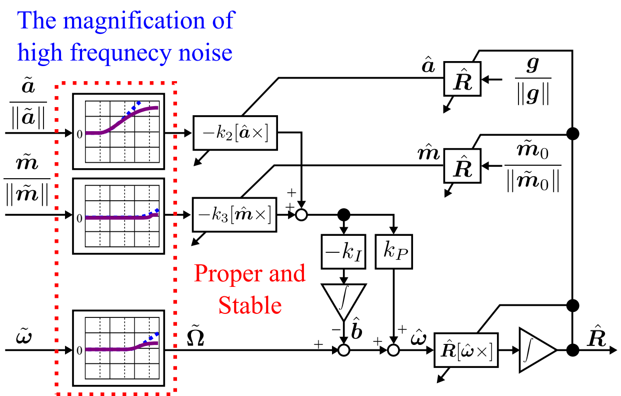
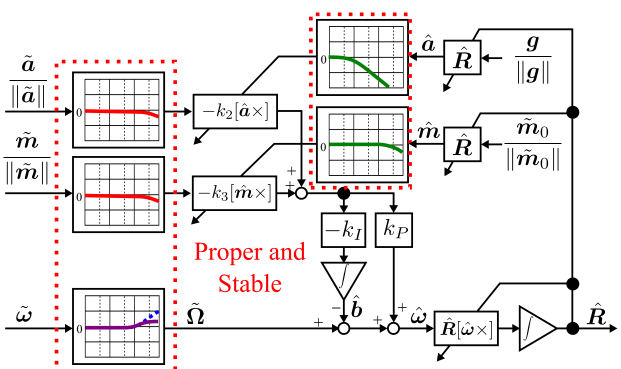


(a) Explicit complementary filter with simply filtering the measurement by the inverse transfer function



(b) Explicit complementary filter with filtering the measurement by the inverse transfer function which is made proper and stable by the relaxation of the complementary condition



(c) The proposed method which only uses the proper and stable transfer function by the relaxed complementary condition and the sensor dynamics filtering of the estimate fed back to the error computation

Fig. 1. Nonlinear complementary filter with the sensor dynamics compensation for the attitude estimation

that of the sensor dynamics compensation is proposed. In order to realize the dynamics compensation only by the proper and stable transfer function, the proposed method is designed based on the following two ideas. The first idea is the relaxation of the complementary condition based on the assumption that the entire transfer function is proper and stable in the entire frequency domain as long as it strictly satisfy the complementary condition only in the target frequency domain (Fig.1(b)). The second is to filter not the measured vector by the inverse function but the estimated vector fed back to the error computation by the nominal sensor dynamics (Fig. 1(c)). In particular, the second idea is effective in preventing the magnification of high frequency noise included in the inclinometer and the magnetometer.

II. The Complementary Filter for Attitude Estimation

The complementary filter estimates the original signal from the fusion of some signals extracted by the filter which satisfies the complementary condition in the frequency domain. Let $\boldsymbol{F}_i(s)$ be the frequency filter for i-th sensor output $\boldsymbol{X}_i(s)$. The original signal $\boldsymbol{Y}(s)$ is estimated as

$$\hat{\boldsymbol{Y}}(s) = \sum_{i=1}^{n} \boldsymbol{F}_i(s)\boldsymbol{X}_i(s), \tag{1}$$

where n means the total number of sensors. $\hat{*}$ denotes the estimate of $*$ hereafter. $\boldsymbol{F}_i(s)$ satisfies the complementary condition, namely,

$$\sum_{i=1}^{n} \boldsymbol{F}_i(s) = \mathbf{1}, \tag{2}$$

where $\mathbf{1}$ is the identity matrix.

Let us consider the case that the inclination angle θ is estimated by the rate gyroscope and the inclinometer. Based on Eq. (1), θ is estimated as

$$\hat{\theta} = F_1(s)\frac{1}{s}\tilde{\omega} + F_2(s)\tilde{\theta}. \tag{3}$$

where $\tilde{\omega}$ and $\tilde{\theta}$ are the measured angular velocity and the measured angle, respectively, and is represented as

$$\tilde{\omega} = G_1(s)s\theta + w_1, \tag{4}$$

$$\tilde{\theta} = G_2(s)\theta + w_2, \tag{5}$$

$G_1(s)$ and $G_2(s)$ are the dynamics of the rate gyroscope and the inclinometer, w_1 and w_2 are the noise of the rate gyroscope and the inclinometer, $F_1(s)$ and $F_2(s)$ are the filter for $\tilde{\omega}$ and $\tilde{\theta}$, respectively. $F_1(s)$ is designed as a high-pass filter (HPF) since $\frac{1}{s}\tilde{\omega}$ is reliable in the high frequency domain. On the other hand, $F_2(s)$ is designed as a low-pass filter (LPF) based on the complementary condition.

In the case of that $\tilde{\omega}$ includes the bias offset, one option to design the filter is to design $F_1(s)\frac{1}{s}$ as a band-pass filter, namely,

$$\hat{\theta} = \frac{s^2}{k_I + k_P s + s^2}\frac{1}{s}\tilde{\omega} + \frac{k_I + k_P s}{k_I + k_P s + s^2}\tilde{\theta}, \tag{6}$$

where k_I and k_P are the design parameter. Let $\hat{b}$ be the estimate of the gyroscope bias b, the differential equations, which are equivalent to Eq. (6), are written as

$$\dot{\hat{\theta}} = \tilde{\omega} - \hat{b} + k_P\omega_{\mathrm{mes}}, \tag{7}$$

$$\dot{\hat{b}} = -k_I\omega_{\mathrm{mes}}, \tag{8}$$

$$\omega_{\mathrm{mes}} = \tilde{\theta} - \hat{\theta}. \tag{9}$$

Eqs. (7)-(9) mean that the attitude is estimated by integrating the measured angular velocity corrected by the error between the measurement and the estimate. Some nonlinear complementary filter [7], [8] are the expansion of Eqs. (7)-(9). In particular, the explicit complementary filter (ECF) proposed by Mahony et al. [8] has a simple computation of the error between the measurement and the estimate, and is represented as

$$\dot{\hat{\boldsymbol{R}}} = \hat{\boldsymbol{R}} \left[\left(\tilde{\boldsymbol{\omega}} - \hat{\boldsymbol{b}} + k_P \boldsymbol{\omega}_{\rm mes} \right) \times \right] \tag{10}$$

$$\dot{\hat{\boldsymbol{b}}} = -k_I \boldsymbol{\omega}_{\rm mes} \tag{11}$$

$$\boldsymbol{\omega}_{\rm mes} = \sum_{i=2}^{n} k_i \tilde{\boldsymbol{v}}_i \times \hat{\boldsymbol{v}}_i \tag{12}$$

where $\boldsymbol{R} \in SO(3)$ is the attitude matrix, $\boldsymbol{b} \in \mathbb{R}^3$ is the gyroscope bias, $\tilde{\boldsymbol{\omega}} \in \mathbb{R}^3$ is the measured angular velocity, and $\tilde{\boldsymbol{v}}_i \in \mathbb{R}^3$ is the vector measurement such as the direction of the gravity and the geomagnetism. Note that the vector measurement is linearly treated in Eq. (12). $[*\times]$ is a skew-symmetric matrix which means the cross product by $*$. $\hat{\boldsymbol{v}}_i \in \mathbb{R}^3$ is the estimate of the vector measurement, for instance, the estimated direction of the gravity and the geomagnetism are written as

$$\hat{\boldsymbol{v}}_2 = \hat{\boldsymbol{R}}^{\rm T} \frac{\boldsymbol{g}}{\|\boldsymbol{g}\|} \tag{13}$$

$$\hat{\boldsymbol{v}}_3 = \hat{\boldsymbol{R}}^{\rm T} \frac{\boldsymbol{m}_0}{\|\boldsymbol{m}_0\|} \tag{14}$$

where $\boldsymbol{g} = [0\ 0\ g]^{\rm T}$ and $\boldsymbol{m}_0$ are the acceleration due to the gravity and the direction of the magnetism at the initial time, respectively. k_i is the weight for $\tilde{\boldsymbol{v}}_i \times \hat{\boldsymbol{v}}_i$. ECF assumes that the sensor dynamics can be ignored, but those dynamics affect the estimation accuracy in fact.

Although the attitude is represented by the 1D angle, Baerveldt et al. [12] compensated the sensor dynamics as

$$\hat{\theta} = F_1(s) \frac{1}{s} \tilde{G}_1^{-1}(s) \tilde{\omega} + F_2(s) \tilde{G}_2^{-1}(s) \tilde{\theta}. \tag{15}$$

where $\tilde{G}_1(s)$ and $\tilde{G}_2(s)$ are the nominal transfer function of the rate gyroscope and the inclinometer, respectively. However, it is difficult to simply apply it to 3D attitude estimation due to that the nonlinearity of 3D attitude affects the sensor dynamics. For this problem, the authors [9] proposed a 3D attitude estimator which combines the output of the rate gyroscope, the inclinometer, and the magnetometer. The authors' previous method deals with the nonlinear sensor dynamics by dividing it into the nonlinear and linear part. The former part is the nonlinear coordinate transformation which can be derived from the relationship between the sensor output and the attitude representation theoretically. The latter part is the linear transfer function which can be identified by the frequency response. The inverse transfer function tends to be non-proper and unstable, so that the designed filter is inserted between the nonlinear and linear part to make the transfer function proper and stable. However, the disturbance included in the magnetometer's output strongly affects the azimuth angle computed by the coordinate transformation.

III. The Nonlinear Complementary Filter with Sensor Dynamics Compensation

To summarize the above, ECF is affected by the sensor dynamics but it can linearly treat the magnetism measurement without the transformation. On the other hand, the authors' previous method can compensate the sensor dynamics but its azimuth angle computation is disturbed by the magnetometer noise. Therefore, this paper aims to improve the estimation accuracy by fusing the idea of ECF and that of the dynamics compensation. Focusing on the similarity of Eqs. (7)-(9) and ECF, the proposed filter is designed to expand 1D attitude estimator to 3D attitude estimator.

As shown in Fig. 1(a), the simplest idea to compensate the sensor dynamics is to use the inverse transfer function directly as

$$\dot{\hat{\theta}} = \tilde{\Omega} - \hat{b} + k_P \omega_{\rm mes}, \tag{16}$$

$$\dot{\hat{b}} = -k_I \omega_{\rm mes}, \tag{17}$$

$$\tilde{\Omega} = \tilde{G}_1^{-1}(s) \tilde{\omega}, \tag{18}$$

$$\omega_{\rm mes} = \tilde{G}_2^{-1}(s) \tilde{\theta} - \hat{\theta}. \tag{19}$$

However, the nominal inverse transfer functions $\tilde{G}_1^{-1}(s)$ and $\tilde{G}_2^{-1}(s)$ tend to be non-proper and unstable. In order to use only the proper and stable transfer function, we focus on the complementary condition. The entire transfer function $F_0(s)$ must satisfy the complementary condition Eq. (2) in the target frequency domain, but it do not have to do in the entire frequency domain. Namely, $F_0(s)$ is allowed to be the proper and stable one in the entire frequency domain as long as $F_0(s) = 1$ in the target frequency domain. Therefore, the complementary condition is relaxed as

$$\sum_{i=1}^{n} F_i(s) = F_0(s). \tag{20}$$

From this idea, $\tilde{G}_1^{-1}(s)$ and $\tilde{G}_2^{-1}(s)$ in Eqs. (18) and (19) are replaced by the proper and stable transfer function $F_0(s) G_1^{-1}(s)$ and $F_0(s) G_2^{-1}(s)$ as

$$\tilde{\Omega} = F_0(s) \tilde{G}_1^{-1}(s) \tilde{\omega}, \tag{21}$$

$$\omega_{\rm mes} = F_0(s) \tilde{G}_2^{-1}(s) \tilde{\theta} - \hat{\theta}. \tag{22}$$

However, when the cut-off frequency of $G_2^{-1}(s)$ is much lower than that of $F_0(s)$, the magnification of the high frequency noise is not negligible (Fig.1(b)). In order to avoid the magnification, $\omega_{\rm mes}$ is computed based on the filtering of not the measurement by the inverse transfer function but the estimate by the nominal transfer function, namely,

$$\omega_{\rm mes} = F_0(s) \tilde{\theta} - \tilde{G}_2(s) \hat{\theta}. \tag{23}$$

By substituting Eq. (5) to Eq. (23), it is easily verified that the sensor noise w_2 is filtered only by $F_0(s)$. The designed 1D attitude estimator consisting of Eqs. (16), (17), (21) and

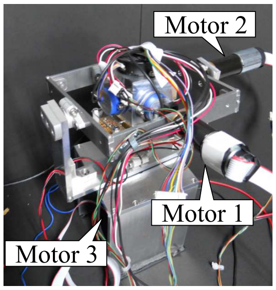


Fig. 2. The experimental machine

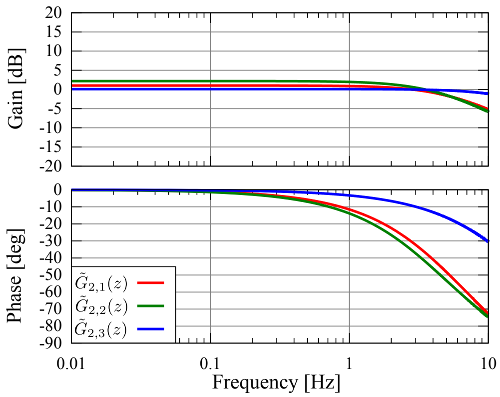


Fig. 3. The frequency response of $\tilde{\boldsymbol{G}}_2(z)$

(23) is equivalent to the following complementary filter:

$$\hat{\theta} = F_0 \frac{s^2}{\tilde{G}_2(s)k_I + \tilde{G}_2(s)k_P s + s^2} \frac{1}{s}\tilde{\omega} + F_0(s)\frac{\tilde{G}_2(s)k_I + \tilde{G}_2(s)k_P s}{\tilde{G}_2(s)k_I + \tilde{G}_2(s)k_P s + s^2}\tilde{G}_2^{-1}(s)\tilde{\theta}. \quad (24)$$

As well as Eqs. (4) and (5), $\tilde{\boldsymbol{\omega}}$ and $\tilde{\boldsymbol{v}}_i$ $(i = 2, \cdots, n)$ are represented as

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{G}_1(s)\boldsymbol{\omega} + \boldsymbol{w}_1, \quad (25)$$
$$\tilde{\boldsymbol{v}}_i = \boldsymbol{G}_i(s)\boldsymbol{v}_i + \boldsymbol{w}_i, \quad (i = 2, \cdots, n), \quad (26)$$

where $\boldsymbol{w}_i$ $(i = 1, \cdots, n)$ is the white noise, $\boldsymbol{G}_1(s)$ and $\boldsymbol{G}_i(s)$ $(i = 2, \cdots, n)$ are the transfer function of the rate gyroscope and i-th sensor which outputs the measurement vector, respectively. Finally, 1D attitude estimator is expanded to 3D attitude estimator as

$$\dot{\hat{\boldsymbol{R}}} = \hat{\boldsymbol{R}}\left[\left(\tilde{\boldsymbol{\Omega}} - \hat{\boldsymbol{b}} + k_P\boldsymbol{\omega}_{\mathrm{mes}}\right)\times\right], \quad (27)$$
$$\dot{\hat{\boldsymbol{b}}} = -k_I\boldsymbol{\omega}_{\mathrm{mes}}, \quad (28)$$
$$\tilde{\boldsymbol{\Omega}} = \boldsymbol{F}_0(s)\tilde{\boldsymbol{G}}_1^{-1}(s)\tilde{\boldsymbol{\omega}}, \quad (29)$$
$$\boldsymbol{\omega}_{\mathrm{mes}} = \sum_{i=2}^{n} k_i\left(\boldsymbol{F}_0(s)\tilde{\boldsymbol{v}}_i\right) \times \left(\tilde{\boldsymbol{G}}_i(s)\hat{\boldsymbol{v}}_i\right), \quad (30)$$

where $\boldsymbol{F}_0(s)$ is the entire transfer function, $\tilde{\boldsymbol{G}}_1(s)$ and $\tilde{\boldsymbol{G}}_i(s)$ $(i = 2, \cdots, n)$ are the identified nominal transfer function of the rate gyroscope and i-th sensor which outputs the measurement vector, respectively. Eq. (29) indicates that $\boldsymbol{F}_0(s)$ is designed to make $\boldsymbol{F}_0(s)\tilde{\boldsymbol{G}}_1^{-1}(s)$ proper and stable.

The quaternion form of the proposed method is shown in Appendix A.

IV. Evaluation by the Experiment

A. Set up

The validity of the proposed method is evaluated by the experiment. The overview of the experimental machine is shown in Fig. 2. It is designed based on the gimbal mechanism and can rotate three axes independently. All axes are orthogonal to each other when the angle of each axis is equal to zero. As well as the authors' previous work [9], three 1-axis rate gyroscope CRS07-11S (Silicon Sensing Systems Japan), a 2-axis inclinometer X3M (US Digital), and a 3-axis magnetometer AMI304 (Aichi Micro Intelligent) are employed as the rate gyroscope, the inclinometer, and the magnetometer, respectively.

B. The identification of the sensor dynamics

In order to implement the proposed filter, it is necessary to identify the sensor dynamics of each sensor. The nominal dynamics of the rate gyroscope and the magnetometer have already obtained in the authors' previous work [9], so that this paper uses the following transfer function as the nominal one of the rate gyroscope $\tilde{\boldsymbol{G}}_1(s)$ and the magnetometer $\tilde{\boldsymbol{G}}_3(s)$,

$$\tilde{\boldsymbol{G}}_1(s) = \begin{bmatrix} \frac{1.035686}{1+0.004112s} & \frac{-0.025885}{1+0.004112s} & \frac{0.005136}{1+0.004112s} \\ \frac{0.034362}{1+0.004177s} & \frac{1.070075}{1+0.004177s} & \frac{-0.009853}{1+0.004177s} \\ \frac{-0.038275}{1+0.004858s} & \frac{0.029495}{1+0.004858s} & \frac{1.075213}{1+0.004858s} \end{bmatrix}, \quad (31)$$
$$\tilde{\boldsymbol{G}}_3(s) = \mathrm{diag}\{1.0,\ 1.048,\ 0.980\}, \quad (32)$$

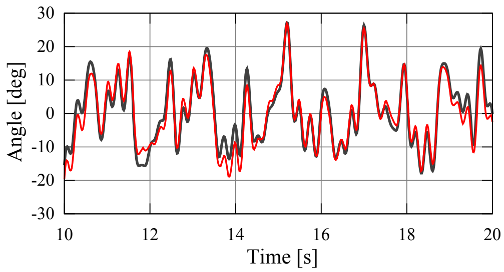
where $\mathrm{diag}\{d_1, \cdots, d_n\}$ means the diagonal matrix which diagonal components are $d_1, \cdots d_n$.

On the other hand, since the dynamics of the gravity direction computed from the inclinometer's output have not obtained yet, it is necessary to identify it afresh. From the viewpoint of the implementation, this paper identifies it in the discrete time based on auto-regressive exogenous (ARX) model. Let ΔT the sampling time, ARX model with respect to the gravity direction is represented as

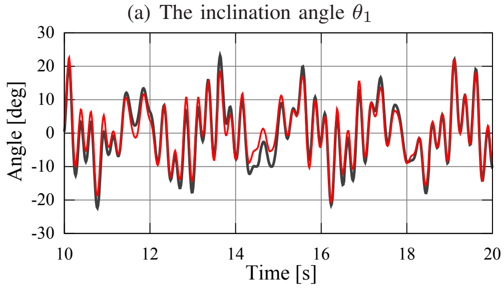
$$\boldsymbol{A}(z)\tilde{\boldsymbol{v}}_{2,k} = \boldsymbol{B}(z)\boldsymbol{v}_{2,k} + \boldsymbol{w}_2, \quad (33)$$

where the subscript k means the index of the time $k\Delta T$. $\boldsymbol{A}(z)$ and $\boldsymbol{B}(z)$ are the matrices which component is represented by the polynomials with respect to the shift operator z, and correspond to the denominator and numerator of the transfer function, respectively. This paper assumed that $\boldsymbol{A}(z)$ and $\boldsymbol{B}(z)$ are the diagonal matrices. $\boldsymbol{v}_2$ and $\tilde{\boldsymbol{v}}_2$ are the true gravity direction and the gravity direction from the inclinometer, respectively. In the identification, the gravity direction from encoders attached on the experimental machine is substituted for $\boldsymbol{v}_2$. The identification result is written as

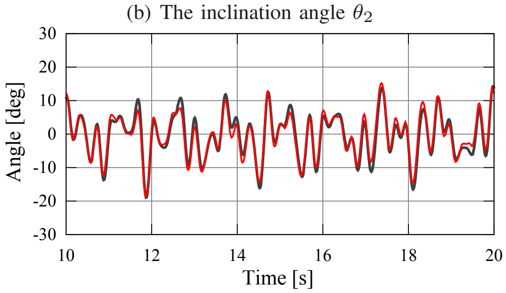
$$\tilde{\boldsymbol{G}}_2(z) = \mathrm{diag}\{\tilde{G}_{2,1}(z),\ \tilde{G}_{2,2}(z),\ \tilde{G}_{2,3}(z)\}, \quad (34)$$



(a) The inclination angle θ_1

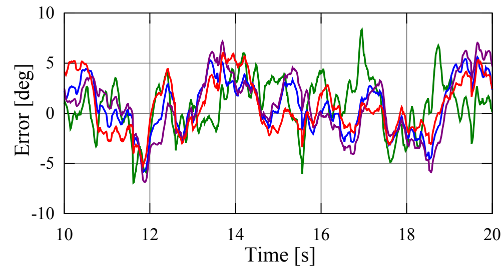


(b) The inclination angle θ_2

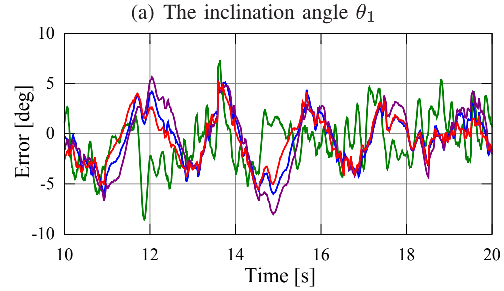


(c) The azimuth angle ψ

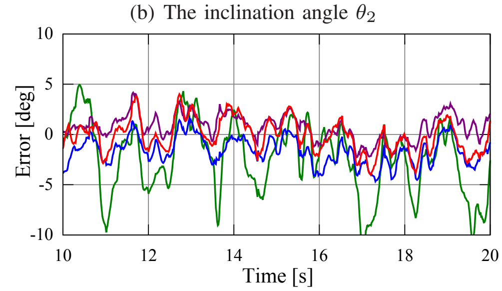
Fig. 4. The enlarged view of an example of the estimation result by the proposed method from 10.0[s] to 20.0[s]. (Gray line is the ground truth, and red line is the proposed method.)



(a) The inclination angle θ_1



(b) The inclination angle θ_2



(c) The azimuth angle ψ

Fig. 5. The enlarged view of an example of the estimation errors from 10.0[s] to 20.0[s]. (Red line is the proposed method, green line is the authors' previous method, blue line is ECF, and purple line is Pro. wo. EVF.)

where

$$\tilde{G}_{2,1}(z) = \frac{-0.0789 + 0.0094z^{-1} + 0.0963z^{-2}}{1.0 - 0.5108z^{-1} - 0.4653z^{-2}}, \quad (35)$$

$$\tilde{G}_{2,2}(z) = \frac{-0.0471 + 0.0086z^{-1} + 0.0629z^{-2}}{1.0 - 0.5039z^{-1} - 0.4771z^{-2}}, \quad (36)$$

$$\tilde{G}_{2,3}(z) = \frac{0.0184 + 0.0251z^{-1} + 0.0333z^{-2}}{1.0 - 0.4619z^{-1} - 0.4623z^{-2}}. \quad (37)$$

Fig. 3 shows the frequency response of $\tilde{\boldsymbol{G}}_2(z)$. In the frequency responses of $\tilde{G}_{2,1}(z)$ and $\tilde{G}_{2,2}(z)$, the lag occurs over about 0.1[Hz].

C. Experimental result

The following rotations are input to the motor of the test bed as the fast and irregular motion:

$$q_i(t) = \sum_{j=1}^{5} \left(a_{ij}\sin(2\pi f_{ij}t) + b_{ij}\cos(2\pi f_{ij}t)\right), \ (i = 1, 2, 3),$$

where a_{ij}[deg] and b_{ij}[deg] are the amplitude of the sine and cosine wave, respectively, and are determined randomly. In order to spread the frequency f_{ij} on the frequency domain under 5[Hz], f_{ij} is determined as

$$f_{ij} \sim j + 0.1DU(10) + 0.01DU(10), \quad (38)$$

where $DU(N)$ means a discrete uniform distribution which range is from 0 to N. The sampling time and the total time are set to 3[ms] and 30[s], respectively.

In the experiment, the following methods are compared:

- The authors' previous method [9] (Previous)
- Explicit complementary filter proposed by Mahony et al.[8] (ECF)
- The proposed method without filtering the estimated vector by the nominal dynamics (Pro. wo. EVF)
- The proposed method (Proposed)

Instead of Eq. (30), Pro. wo. EVF uses the following equation:

$$\boldsymbol{\omega}_{\mathrm{mes}} = \sum_{i=2}^{n} k_i \left(\boldsymbol{F}_0(s)\tilde{\boldsymbol{G}}_i^{-1}(s)\tilde{\boldsymbol{v}}_i\right) \times \hat{\boldsymbol{v}}_i. \quad (39)$$

$\boldsymbol{F}_0$ in Pro. wo. EVF and Proposed are designed as

$$\boldsymbol{F}_0 = \frac{2\pi f_0}{2\pi f_0 + s}\mathbf{1}, \quad (40)$$

where f_0 is the cut-off frequency and is determined such that f_0 is about three time of the cut-off frequency of $\boldsymbol{G}_1(s)$, namely, $f_0 = 90.0$[Hz]. All transfer functions used in Pro.

TABLE I

ROOT-MEAN SQUARE ERRORS OF THE ESTIMATED ANGLES

Method	Inclination		Azimuth
	θ_1 [deg]	θ_2 [deg]	ϕ [deg]
Previous	2.94	2.38	4.38
ECF	3.07	2.34	3.05
Pro. wo. EVF	4.01	3.05	3.62
Proposed	2.69	1.99	2.81

wo. EVF and Proposed were discretized by the bilinear transformation. In order to exclude the effect of tuning from the estimation, we tuned k_P and k_I of ECF, Pro. wo. EVF and Proposed 80,000 times under the condition that both k_2 and k_3 of them are fixed as 0.5, and selected the set of k_P and k_I which outputs the best result for each data obtained by experiments. Similarly, parameters of Previous are also selected.

Table I shows the root-mean-square error (RMSE) of the result. RMSE of each angle is computed as

$$RMSE_* = \sqrt{\frac{1}{N_T N_D}\sum_{i=1}^{N_T}\sum_{j=1}^{N_D} e_{*,ij}^2},\ (* = \theta_1,\ \theta_2,\ \psi),$$

where N_T and N_D are total number of trials and data, respectively. In this paper, they are set as $N_T = 4,\ N_D = 3000$. $e_{\theta_1,ij}$, $e_{\theta_2,ij}$ and $e_{\psi,ij}$ are the error of θ_1, θ_2 and ψ at $j\Delta T$ in ith trial, respectively. An example of the estimation result by the proposed method is plotted in Fig. 4, and that of the estimation errors is plotted in Fig. 5. From the result, the inclination angle estimated by Previous is better than ECF, but its azimuth angle estimation is the worst compared with the other method due to the coordinate transformation. On the other hand, the estimation accuracy of the inclination angle estimated by Pro. wo. EVF is lower than that of ECF. This result shows that the magnification of the high frequency noise is more effective to the estimation accuracy than the dynamics compensation. Compared with other methods, the proposed method shows the best estimation result in all angles. Therefore, the combination of correcting the measured angular velocity by the vector measurement and the dynamics compensation is valid for the attitude estimation, and it is verified that the combination can be implemented by the relaxed complementary condition and the estimated vector filtered by the sensor dynamics.

V. CONCLUSION

This paper proposes a novel nonlinear complementary filter for 3D attitude estimation of the fast and irregular attitude variation. The proposed method is designed based on the scheme of ECF, and compensates the sensor dynamics by only using the proper and stable transfer functions. Focusing on the condition of the entire transfer function, the inverse transfer function, which tends to be non-proper and unstable, is made proper and stable by the fusing it with the entire transfer function. The usage of the inverse transfer function causes the magnification of the high frequency noise, so that the estimate fed back to the error computation is filtered by the sensor dynamics instead of filtering the measurement by the inverse function. Through the experiment for the fast and irregular motion under 5[Hz], it is verified that the proposed method can improve the estimation accuracy.

APPENDIX

A. *The proposed method represented by the quaternion*

In order to avoid the singularity problem, the quaternion is often used as the attitude, so that this section represents the proposed method by the quaternion. Let $\boldsymbol{q}$ be the quaternion corresponding to the attitude matrix $\boldsymbol{R}$, the quaternion differentiation is represented as

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q} \otimes \boldsymbol{\psi}\left(\boldsymbol{\omega}\right) \tag{41}$$

where the operator $\otimes$ denotes the quaternion product, and $\boldsymbol{\psi}(*)$ means the quaternion corresponds to a 3D vector $*$. From Eq. (41), the quaternion form of the proposed method consists of Eqs. (28)-(30) and the following equation.

$$\dot{\hat{\boldsymbol{q}}} = \frac{1}{2}\hat{\boldsymbol{q}} \otimes \boldsymbol{\psi}\left(\tilde{\boldsymbol{\Omega}} - \hat{\boldsymbol{b}} + k_P \boldsymbol{\omega}_{\mathrm{mes}}\right). \tag{42}$$

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