# A Dual-stage Complementary Filter for Dead Reckoning of a Biped Robot via Estimated Contact Point 

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#### Abstract

This paper proposes a novel technique of dead reckoning for biped robots. The trunk position of a robot with respect to the inertial frame is estimated only using internal sensors in a short interval, so that it is available for high-rate feedback control. It is a complementary filter in which (1) the low-frequency component of the motion of the trunk is inversely computed from the relative motion of the contact point with respect to the trunk, where the rotation and rolling of the support foot about the contact point are taken into account, (2) the high-frequency component of that is estimated by doubleintegral of acceleration, and (3) the crossover frequency to combine those estimations is automatically adjusted based on the ground reaction force. The efficacy of the proposed method was verified through some simulations.


## I. INTRODUCTION

Estimation of the current position is a crucial issue to control mobile robots such as wheeled robots and legged robots. Particularly, fast and accurate estimation is required for a reliable control to enable dynamic motions. External sensors such as cameras and laser range finders, which are commonly used for the global localization[1], [2], [3], are hardly available for this purpose due to their slow sampling rate. A possible solution to this problem is the estimation based on the information only from internal sensors with high sampling rates, namely, dead reckoning.

The dead reckoning is also a common technique in the field of wheeled robots[4]. The total traveling distance and displacement of a robot can be estimated based on the information from motion profiles of each wheel provided by rotary encoders. Some techniques to combine the dead reckoning and the global localization using external sensors have also been proposed[5], [6], [7]. On the other hand, in the field of legged robots, the motion of legs are often utilized instead of that of wheels [8].The relative motion of support foot, which is hopefully stationary with respect to the inertial frame, is computed based on the kinematics, and then the motion of the trunk with respect to the inertial frame is inversely estimated. The above assumption, however, breaks when the support foot rotates about the contact point or rolls on the ground, and in such situations, the estimation accuracy is necessarily degraded. Another option is doubleintegral of acceleration[9] measured by an accelerometer. It

[^0]easily suffers from the accumulation of errors which mainly arises due to the drift of signal. Bloesch et al.[10] proposed a nonlinear Kalman filter which combines the above two approaches to compensate each other. Chilian et al.[11] also composed an information filter in which a vision sensor was added to the above two. They can still hardly care the situations where the support foot rolls or rotates, in addition to a burden of tuning the parameters of the system designed in the time domain.

Basically, the estimation based on the kinematics computation is reliable in the low-frequency domain since the motion of the support foot with respect to the ground is usually slow, while the double-integral of acceleration is in the high-frequency domain for the accelerometer detects quick movements. Based on this, we proposed a dead reckoning which combines them in a complementary manner in terms of properties in the frequency domain[12]. It is an application of the complementary filter[13]. In our previous method, the estimation based on the kinematics computation is also improved, in which we assume that minimum velocity point (MVP) with respect to the inertial frame can substitute the contact point. It works even in situations when the support foot rolls or rotates on the ground. It is easily tuned since the parameter is only the crossover frequency, namely, the cut-off frequency of both the low-pass filter (LPF) and the high-pass filter (HPF). A technique includes an automatic adjustment of the crossover frequency in accordance with the ground reaction force, since the estimation based on the kinematics computation is more reliable when the support foot firmly contacts on the ground. However, while the position estimator was designed under the assumption that the foot moves. the velocity estimator used the velocity in the case of that the support foot is fixed on the ground. This paper resolves the mismatch by feeding back the differential of the position estimation to the velocity estimator.

## II. DEAD RECKONING BASED ON THE ESTIMATED MINIMUM VELOCITY POINT

A legged robot moves by alternating its supporting leg. The forward kinematics between the supporting foot and the trunk is written as

$$
\begin{align*}
\boldsymbol{p}_{S} & =\boldsymbol{p}_{0}+\boldsymbol{R}_{0}{ }^{0} \boldsymbol{p}_{S}  \tag{1}\\
\boldsymbol{v}_{S} & =\boldsymbol{v}_{0}+\left[\boldsymbol{\omega}_{0} \times\right] \boldsymbol{R}_{0}{ }^{0} \boldsymbol{p}_{S}+\boldsymbol{R}_{0}{ }^{0} \boldsymbol{v}_{S},  \tag{2}\\
\boldsymbol{R}_{S} & =\boldsymbol{R}_{0}{ }^{0} \boldsymbol{R}_{S}  \tag{3}\\
\boldsymbol{\omega}_{S} & =\boldsymbol{\omega}_{0}+\boldsymbol{R}_{0}{ }^{0} \boldsymbol{\omega}_{S}, \tag{4}
\end{align*}
$$

where the subscripts $S$ and 0 mean the support foot frame $\Sigma_{S}$ and the body frame $\Sigma_{0}$, respectively. ${ }^{{ }^{1}} \boldsymbol{p}_{*_{2}},{ }^{{ }^{*}} \boldsymbol{v}_{*_{2}},{ }^{{ }^{*}} \boldsymbol{R}_{*_{2}}$


Fig. 1. The proposed dead reckoning (a)the overview of proposed dead reckoning(the red box is the estimator, the green and purple box denote KC and DIA, respectively, and the blue arrow is the reaction force). (b)the detail of "Position Update"(the red box denotes MVP estimator)
and ${ }^{{ }^{*}} \boldsymbol{\omega}_{*_{2}}$ are the position, velocity, attitude and angular velocity of $\Sigma_{*_{2}}$ with respect to $\Sigma_{*_{1}}$, respectively. In the case of the inertial frame $\Sigma$, the corresponding subscripts are omitted. The values of $\Sigma_{*}$ with respect to $\Sigma_{0}$ can be calculated from joint angles $\boldsymbol{q}$ and their differential $\dot{\boldsymbol{q}}$. Also, $\boldsymbol{R}_{0}$ and $\boldsymbol{\omega}_{0}$ are able to be obtained by attitude estimator, e.g. by our previous work[14].

When the support foot is assumed to be fixed on the ground during one step, the trunk position with respect to the inertial frame can be estimated by the inverse computation based on the kinematics from the relative motion of the support foot with respect to the trunk (KCSF)[8]. However, its accuracy is degraded when the support foot rotates around a contact point or rolls on the ground. Double-integral of acceleration (DIA) is another idea[9]. It has a high reliability for fast movements comparatively, but it consequently suffers from the accumulation of errors caused by low frequency signals with the integration. In order to improve the accuracy, Bloesch et al.[10] and Chilian et al.[11] combine them by a nonlinear Kalman filter and an information filter, respectively. However, those filters need the statistical properties of signals, which causes the difficulty to tune the parameters of filters.

For those problem, we propose a novel technique of dead reckoning for biped robots. It is basically a complementary filter which is designed in the frequency domain[13]. Fig. 1 shows the proposed dead reckoning. As indicated by the red box in Fig. 1(a), the proposed has a dual-stage complementary filter consisting of the position estimator and the velocity estimator. The position estimator combines DIA with the trunk position calculated by the inverse computation based on the kinematics (KC) by a complementary filter. The former signal has high accuracy in the high frequency domain, so that we employ HPF which includes the doubledifferential operator to cancel the double-integral operator. In contrast, the latter is reliable in the low frequency domain, so that it is filtered by LPF designed complementarily. Similarly, the velocity estimator combines the differential of the position estimation with the integral of the acceleration in a complementary manner, instead of the velocity obtained
by KCSF[12]. The result of that estimator is used to improve the estimation by KC. In our method, a point named MVP which has the minimum velocity with respect to the inertial frame on the foot is used as the basis of KC , instead of the fixed support foot. MVP is calculated in the block named "MVP Estimator" which is included in "Position Update" and indicated by the red box in Fig. 1(b). Although that point is not uniquely determined when the angular velocity is nearly equal to zero, we resolve that problem by using the maximum likelihood estimation. Namely, MVP is computed as the minimizer of an evaluation function consisting of that about not only the MVP's velocity with respect to the inertial frame but also the variation of MVP on the foot. Then, the foot position is updated via MVP and the trunk position is calculated by KC. In order to obtain the high reliable trunk position, each trunk position from foot is combined based on the reaction force. Also, the relative reliability of each signal is assumed to vary with the contact condition, so that the crossover frequency of those estimator are adaptively designed in accordance with the reaction force as indicated by blue arrow in Fig. 1(a). Those details are shown in the later.

## III. THE COMPUTATION OF THE ESTIMATED MINIMUM VELOCITY POINT[12]

The supporting foot of biped robot is either the left foot or the right foot, so that we use a subscript $L$ or $R$ as the left foot frame $\Sigma_{L}$ and the right foot frame $\Sigma_{R}$, respectively, instead of the support foot frame $\Sigma_{S}$ hereafter. From the viewpoint of implementation, variables are represented in a discretized way with the sampling time $\Delta T$. For example, the variable $*$ at the time $(k-1) \Delta T$ is denoted by $*[k-1]$. Exceptionally, a variable at the time $k \Delta T$ is written simply as $*$ without $[k]$ for the reader's convenience. Also, $\mathbf{0} \in \mathbb{R}^{3}$ and $1 \in \mathbb{R}^{3 \times 3}$ denote the zero vector and the identity matrix.
The position and velocity of a point on the left foot frame with respect to the inertial frame is calculated as

$$
\begin{align*}
\boldsymbol{p}_{L, m} & =\boldsymbol{p}_{L}+\boldsymbol{R}_{L}{ }^{L} \boldsymbol{p}_{L, m}  \tag{5}\\
\boldsymbol{v}_{L, m} & =\boldsymbol{v}_{L}+\left[\boldsymbol{\omega}_{L} \times\right] \boldsymbol{R}_{L}{ }^{L} \boldsymbol{p}_{L, m}+\boldsymbol{R}_{L}{ }^{L} \boldsymbol{v}_{L, m} \tag{6}
\end{align*}
$$



Fig. 2. The update of link position based on MVP
where $\boldsymbol{p}_{L, m}$ and ${ }^{L} \boldsymbol{p}_{L, m}$ are the position of a point with respect to $\Sigma$ and $\Sigma_{L}$, respectively. Similarly, $\boldsymbol{v}_{L, m}$ and ${ }^{L} \boldsymbol{v}_{L, m}$ denote the velocity of the point with respect to those frame, respectively. If we find a point which satisfies $\boldsymbol{v}_{L, m} \simeq \mathbf{0}$, it is possible to update the foot position based on that point and Eqn.(5) as shown in Fig. 2. Suppose that ${ }^{L} \boldsymbol{v}_{L, m} \simeq \mathbf{0}$, Eqn.(6) is rewritten as follows;

$$
\begin{equation*}
\boldsymbol{v}_{L, m} \simeq \boldsymbol{v}_{L}+\left[\boldsymbol{\omega}_{L} \times\right] \boldsymbol{R}_{L}{ }^{L} \boldsymbol{p}_{L, m} \tag{7}
\end{equation*}
$$

The objective of this computation is to obtain ${ }^{L} \boldsymbol{p}_{L, m}$ which is MVP on the left foot, so that we consider an evaluation function based on Eqn.(7) as

$$
\begin{align*}
E_{1} & =\frac{1}{2}\left\|\boldsymbol{v}_{L, m}\right\|^{2} \\
& =\frac{1}{2}\left\|\boldsymbol{v}_{L}+\left[\boldsymbol{\omega}_{L} \times\right] \boldsymbol{R}_{L}{ }^{L} \boldsymbol{p}_{L, m}\right\|^{2} . \tag{8}
\end{align*}
$$

$E_{1}$ has the global minimizer, which satisfies $\left(\frac{\partial E_{1}}{\partial^{L} \boldsymbol{p}_{L, m}}\right)^{\mathrm{T}}=$ $\mathbf{0}$. The general solution is obtained as

$$
\begin{equation*}
{ }^{L} \boldsymbol{p}_{L, m}=\frac{1}{{ }^{L} \boldsymbol{\omega}^{\mathrm{T} L} \boldsymbol{\omega}}\left[{ }^{L} \boldsymbol{\omega} \times\right] \boldsymbol{R}_{L}^{\mathrm{T}} \boldsymbol{v}_{L}+c \frac{{ }^{L} \boldsymbol{\omega}}{\left\|{ }^{L} \boldsymbol{\omega}\right\|}, \tag{9}
\end{equation*}
$$

where $c$ is a constant and ${ }^{L} \boldsymbol{\omega}=\boldsymbol{R}_{L}^{\mathrm{T}} \boldsymbol{\omega}_{L}$. From the viewpoint of the computation, that solution has a serious and unpreferable problem. Namely, when $\boldsymbol{\omega}_{L} \rightarrow \mathbf{0}$, MVP is not unique.

For this problem, we minimize not only an evaluation function related to the velocity of MVP with respect to the inertial frame, but also that related to the velocity on the left foot in the previous step. While the former is the same in above, the latter means the difference between the current MVP ${ }^{i} \boldsymbol{p}_{i, m}$ and the previous MVP ${ }^{i} \boldsymbol{p}_{i, m}[k-1]$. Then, the evaluation function is redefined as

$$
\begin{equation*}
E=E_{1}+\frac{1}{T_{m}^{2}} E_{2} \tag{10}
\end{equation*}
$$

where the second term in the right-hand side means the above new evaluation function, namely,

$$
\begin{align*}
E_{2} & =\frac{1}{2}\left\|^{L} \boldsymbol{v}_{L, m}[k-1] \Delta T\right\|^{2} \\
& \simeq \frac{1}{2}\left\|{ }^{L} \boldsymbol{p}_{L, m}-{ }^{L} \boldsymbol{p}_{L, m}[k-1]\right\|^{2} . \tag{11}
\end{align*}
$$

$T_{m}$ is the positive time constant working as a weight. If $T_{m} \rightarrow 0$, then $E \rightarrow E_{2}$ and ${ }^{L} \boldsymbol{p}_{L, m}$ does not move from the
initial value. On the other hand, if $T_{m} \rightarrow \infty$, then $E \rightarrow E_{1}$ and ${ }^{L} \boldsymbol{p}_{L, m}$ approaches the true MVP but it suffers from the ill-posedness of computation.
$E$ also has the minimizer, which satisfies $\left(\frac{\partial E}{\partial^{L} \boldsymbol{p}_{L, m}}\right)^{\mathrm{T}}=\mathbf{0}$ and is finally obtained as

$$
\begin{equation*}
{ }^{L} \boldsymbol{p}_{L, m}=\boldsymbol{C}_{1, L} \boldsymbol{R}_{L}^{\mathrm{T}} \boldsymbol{v}_{L}+\boldsymbol{C}_{2, L}{ }^{L} \boldsymbol{p}_{L, m}[k-1], \tag{12}
\end{equation*}
$$

where

$$
\begin{align*}
\boldsymbol{C}_{1, L} & =\frac{T_{m}^{2}}{\left\|{ }^{L} \boldsymbol{\omega}\right\|^{2} T_{m}^{2}+1}\left[{ }^{L} \boldsymbol{\omega} \times\right]  \tag{13}\\
\boldsymbol{C}_{2, L} & =\frac{T_{m}^{2}}{\left\|{ }^{L} \boldsymbol{\omega}\right\|^{2} T_{m}^{2}+1}\left({ }^{L} \boldsymbol{\omega}^{L} \boldsymbol{\omega}^{\mathrm{T}}+\frac{1}{T_{m}^{2}} \mathbf{1}\right) \tag{14}
\end{align*}
$$

Likewise, ${ }^{R} \boldsymbol{p}_{R, m}$ can also be obtained.
MVP is computed under the assumption that $\boldsymbol{v}_{L, m} \simeq \mathbf{0}$, so that the foot's slippage deteriorates the accuracy of MVP.

## IV. IMPLEMENTATION OF THE PROPOSED DEAD RECKONING

## A. Design of the dual-stage complementary filter

1) The position estimator: In order to obtain the estimated trunk position $\hat{\boldsymbol{p}}_{0}$, a complementary filter combines DIA with the trunk position estimated by KC. Here, in order to cancel the double-integral operator $1 / s^{2}$ which is included in DIA and makes the transfer function unstable, the filter includes the double-differential operator $s^{2}$. In addition, each transfer function must be proper, so that we choose a second-order polynomial as the denominator of the filter. Therefore, the position estimator is written as

$$
\begin{equation*}
\hat{\boldsymbol{p}}_{0}=\boldsymbol{H}_{p 1}(z) \boldsymbol{a}_{0}+\boldsymbol{H}_{p 2}(z) \tilde{\boldsymbol{p}}_{0} \tag{15}
\end{equation*}
$$

where $\boldsymbol{a}_{0}$ is the trunk acceleration measured by the accelerometer and $\tilde{\boldsymbol{p}}_{0}$ denotes the trunk position estimated by KC. $\boldsymbol{H}_{p 1}(z)$ and $\boldsymbol{H}_{p 2}(z)$ are the filters which are transformed from the following filters by the bilinear transformation.

$$
\begin{align*}
\frac{1}{s^{2}} \boldsymbol{F}_{p 1}(s) & =\frac{1}{s^{2}} \frac{\left(1 / 2 \pi f_{p}\right)^{2} s^{2}}{\left(1+\left(1 / 2 \pi f_{p}\right) s\right)^{2}}  \tag{16}\\
\boldsymbol{F}_{p 2}(s) & =\mathbf{1}-\boldsymbol{F}_{p 1}(s)=\frac{1+\left(1 / \pi f_{p}\right) s}{\left(1+\left(1 / 2 \pi f_{p}\right) s\right)^{2}} \tag{17}
\end{align*}
$$

where $f_{p}$ is the crossover frequency designed by the reaction force which is described later.
2) The velocity estimator: The trunk velocity with respect to the inertial frame is necessary to calculate the foot velocity used to estimate MVP, but itself is also the object to estimate. For this reason, we estimate it by a complementary filter which combines the integral of the acceleration and the velocity which is reliable in the low frequency domain. One of the latter signal is the trunk velocity calculated by KCSF, the other is the differential of the result of the position estimator. In this paper, the differential of estimated position is likely to be more accurate, so that the velocity estimator is written as

$$
\begin{equation*}
\hat{\boldsymbol{v}}_{0}=\boldsymbol{H}_{v 1}(z) \boldsymbol{a}_{0}+\boldsymbol{H}_{v 2}(z) \hat{\boldsymbol{p}}_{0}, \tag{18}
\end{equation*}
$$

where $\hat{\boldsymbol{v}}_{0}$ is the estimated trunk velocity. $\boldsymbol{H}_{v 1}(z)$ and $\boldsymbol{H}_{v 2}(z)$ are the filters which are transformed from the following filters by the bilinear transformation.

$$
\begin{align*}
\frac{1}{s} \boldsymbol{F}_{v 1}(s) & =\frac{1}{s} \frac{\left(1 / 2 \pi f_{v}\right) s}{1+\left(1 / 2 \pi f_{v}\right) s}  \tag{19}\\
s \boldsymbol{F}_{v 2}(s) & =s\left(\mathbf{1}-\boldsymbol{F}_{v 1}(s)\right)=s \frac{1}{1+\left(1 / 2 \pi f_{v}\right) s} \tag{20}
\end{align*}
$$

where $f_{v}$ is the crossover frequency designed by the reaction force which is described later.

## B. The inverse computation based on the kinematics via MVP

In order to compute MVP, the estimated velocity of the left foot with respect to the inertial frame $\hat{\boldsymbol{v}}_{L}$ is obtained by Eqn.(2).

$$
\begin{equation*}
\hat{\boldsymbol{v}}_{L}=\hat{\boldsymbol{v}}_{0}+\left[\boldsymbol{\omega}_{0} \times\right] \boldsymbol{R}_{0}{ }^{0} \boldsymbol{p}_{L}+\boldsymbol{R}_{0}{ }^{0} \boldsymbol{v}_{L} . \tag{21}
\end{equation*}
$$

By using $\hat{\boldsymbol{v}}_{L}$, MVP is computed by Eqn.(12). Based on the estimated MVP, the left foot position is updated temporarily as shown in Fig. 2,

$$
\begin{equation*}
\tilde{\boldsymbol{p}}_{L}=\hat{\boldsymbol{p}}_{L}[k-1]-\boldsymbol{R}_{L}{ }^{L} \hat{\boldsymbol{p}}_{L, m}+\boldsymbol{R}_{L}[k-1]^{L} \hat{\boldsymbol{p}}_{L, m} . \tag{22}
\end{equation*}
$$

By using Eqn.(1), the trunk position from the left foot $\tilde{\boldsymbol{p}}_{0, L}$ is obtained as

$$
\begin{equation*}
\tilde{\boldsymbol{p}}_{0, L}=\tilde{\boldsymbol{p}}_{L}-\boldsymbol{R}_{0}{ }^{0} \boldsymbol{p}_{L} \tag{23}
\end{equation*}
$$

Likewise, $\hat{\boldsymbol{v}}_{R}, \tilde{\boldsymbol{p}}_{R}$ and $\tilde{\boldsymbol{p}}_{0, R}$ can also be obtained.
In order to obtain a more reliable trunk position, the above two position are combined as

$$
\begin{equation*}
\tilde{\boldsymbol{p}}_{0}=w_{L} \tilde{\boldsymbol{p}}_{0, L}+w_{R} \tilde{\boldsymbol{p}}_{0, R} . \tag{24}
\end{equation*}
$$

Although MVP which has zero velocity is likely to be on the support foot, to determine which is the support foot is difficult due to that biped robots have the double support phase and the sensor output is generally noisy. We assume that the magnitude of the sensor output correlates to what it is the support foot, so that the weight $w_{i}$ is determined by the reaction force of each foot.

$$
\begin{equation*}
w_{i}=\frac{\hat{F}_{z, i}+\epsilon_{F}}{\hat{F}_{z, L}+\hat{F}_{z, R}+2 \epsilon_{F}}, \quad(i=L, R), \tag{25}
\end{equation*}
$$

where $\epsilon_{F}$ is the positive parameter to make the denominator non-zero and $\hat{F}_{z, i}$ is represented as

$$
\hat{F}_{z, i}=\left\{\begin{array}{cc}
0 & \left(F_{z, i}<0\right)  \tag{26}\\
F_{z, i} & \left(0 \leq F_{z, i} \leq M g\right) \quad, \quad(i=L, R) . \\
M g & \left(M g<F_{z, i}\right)
\end{array}\right.
$$

In order to avoid the error accumulation due to the difference between $\tilde{\boldsymbol{p}}_{0}$ and $\tilde{\boldsymbol{p}}_{0, i}$, that difference is fed back to correct the foot position as follows;

$$
\begin{equation*}
\hat{\boldsymbol{p}}_{i}=\tilde{\boldsymbol{p}}_{i}+\left(\tilde{\boldsymbol{p}}_{0}-\tilde{\boldsymbol{p}}_{0, i}\right), \quad(i=L, R) . \tag{27}
\end{equation*}
$$

## C. Design of the crossover frequency by the reaction force

The discussion in the previous section allows various contact condition of the support foot to the ground. The contact condition often varies during the motion. If either foot contacts on the ground, then the estimation by KC has a high reliability. In contrast, when there is no contact, DIA is more reliable. For this reason, we assumed that the relative reliability between KC and DIA changes in accordance with the ground contact. The ground contact is judged by the reaction force and the reliability is able to be denoted by the crossover frequency of filter, so that we adaptively change that frequency of the position estimator based on that force.

$$
f_{p}=\left\{\begin{array}{cc}
f_{p, \text { min }} & \left(F_{z}<0\right)  \tag{28}\\
\hat{f}_{p}\left(F_{z}\right) & \left(0 \leq F_{z} \leq M g\right) \\
f_{p, \max } & \left(M g<F_{z}\right)
\end{array}\right.
$$

where $F_{z}$ is the vertical component of the reaction force, $M$ denotes the mass of the robot and $g$ is the acceleration due to the gravity. $f_{p, \min }$ and $f_{p, \text { max }}$ are the minimum and the maximum crossover frequency of the position estimator, respectively. $\hat{f}_{p}\left(F_{z}\right)$ is the monotone increase function which satisfies the following conditions;

$$
\begin{equation*}
\hat{f}_{p}(0)=f_{p, \min }, \quad \hat{f}_{p}(M g)=f_{p, \max } \tag{29}
\end{equation*}
$$

In this paper, we set $\hat{f}_{p}\left(F_{z}\right)$ as the linear function.
Likewise, $f_{v}$ is also determined.

## V. Simulation

## A. Set up

OpenHRP3[15] was used for the dynamic simulation. As shown in Fig. 3, the robot model has pads at the toe and the heel. Here, we assume that the accelerometer and the gyroscope are attached on the trunk and the force sensor is done on the each ankle. In the simulation, the joint torque $\tau$ was calculated by a PD controller represented as

$$
\begin{equation*}
\boldsymbol{\tau}=\boldsymbol{K}\left(\boldsymbol{q}-{ }^{\mathrm{ref}} \boldsymbol{q}\right)+\boldsymbol{D}\left(\dot{\boldsymbol{q}}-{ }^{\mathrm{ref}} \dot{\boldsymbol{q}}\right) \tag{30}
\end{equation*}
$$

where $\boldsymbol{K}$ and $\boldsymbol{D}$ are the proportional and the differential gains, respectively. The reference of joint angles ${ }^{\text {ref }} \boldsymbol{q}$ and its differential ${ }^{\text {ref }} \dot{\boldsymbol{q}}$ were calculated by the method of Yamamoto et al.[16]. As shown in Fig. 4, the robot walked two steps forward. The positive direction of $x, y$ and $z$ correspond to the forward, leftward and vertical direction of the robot at $0[\mathrm{~s}]$, respectively. Both the static and kinetic friction coefficients between the floor and the robot were set for 1.0.

In the simulations, we compared the following methods;

- KCSF
- DIA filtered by HPF (DIA+HPF)
- The complementary filter which combines KCSF with DIA and does not use MVP (KCSF+DIA)
- The previous method[12] (Previous)
- KC of proposed method $\tilde{p}_{0}$ (KCP)
- The proposed dead reckoning (Proposed)
$f_{*, \min }$ is a small positive value, so that we chose $f_{*, \min }=$ $0.001[\mathrm{~Hz}]$. Since the maximum frequency of the robot's motion was assumed to be about $5[\mathrm{~Hz}]$, we considered that


Fig. 3. Robot model for simulations (a)exterior (b)kinematics (c)side view of the foot


Fig. 4. Snapshots of the simulations
one-tenth of that frequency, namely, $0.5[\mathrm{~Hz}]$, is appropriate for $f_{p, \text { max }}$. Also, the differential of the position estimation is surely accurate when that estimation is reliable, so that $f_{v, \text { max }}$ is set for $5[\mathrm{~Hz}]$. The parameters related to the estimation of MVP and the weighted sum were $T_{m}=0.4$ and $\epsilon_{F}=0.3$, which were the same as the previous method. DIA+HPF used a second-order HPF which form was the same as $\boldsymbol{F}_{p 1}(s)$ in Eqn.(16) and cut-off frequency was set for $0.001[\mathrm{~Hz}]$. The complementary filter of KCSF+DIA was the same as the proposed position estimator. Finally, the parameters of Previous are the same as that shown in [12].

In order to evaluate the effect of the acceleration error, we added the following error $\boldsymbol{w}_{a} \in \mathbb{R}^{3}$ to the true acceleration.

$$
\begin{equation*}
\boldsymbol{w}_{a} \sim \mathcal{N}\left(\boldsymbol{\mu}_{a}, 0.1^{2} \mathbf{1}\right), \boldsymbol{\mu}_{a} \sim \mathcal{N}\left(\mathbf{0}, 0.04^{2} \mathbf{1}\right) \tag{31}
\end{equation*}
$$

where $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the normal distribution with the mean $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma} . \boldsymbol{\mu}_{a}$ was initialized based on the above distribution at the outset of the simulations. The simulations were run ten time with various $\boldsymbol{\mu}_{a}$.

## B. Simulation Result

Table I and Table II show the results of the position estimation and the velocity estimation, respectively. An example of the results are shown in Fig. 5 and Fig. 6, respectively. Proposed and Previous are only plotted in the figures except for that about the error. The result shows that the accuracy of KCSF is degraded due to the motion of the supporting foot. On the other hand, DIA+HPF suffers from the accumulation of the error. KCSF+DIA is able to reduce the error in the velocity estimation compared with KCSF, but its result in the position estimation is nearly equal to KCSF. Compared with those method, Previous, KCP and Proposed can reduce the error in both the position and velocity. Then, it is confirmed that KC via MVP has the advantage to KCSF. Also, from the result of position estimation, KCP is the most accurate than other methods. However, its error of velocity estimation

TABLE I
THE ROOT-MEAN-SQUARE ERROR OF POSITION ESTIMATION

| Method | $x[\mathrm{~mm}]$ | $y[\mathrm{~mm}]$ | $z[\mathrm{~mm}]$ | total $[\mathrm{mm}]$ |
| :---: | :---: | :---: | :---: | :---: |
| KCSF | 22.10 | 6.949 | 36.14 | 65.19 |
| DIA+HPF | 20.10 | 27.26 | 34.91 | 82.27 |
| KCSF+DIA | 19.63 | 5.719 | 39.48 | 64.83 |
| Previous | 9.977 | 9.176 | 19.07 | 38.22 |
| KCP | 4.473 | 7.083 | 7.214 | 18.77 |
| Proposed | 6.588 | 8.333 | 9.641 | 24.56 |

TABLE II
THE ROOT-MEAN-SQUARE ERROR OF ESTIMATED VELOCITY

| Method | $x[\mathrm{~mm} / \mathrm{s}]$ | $y[\mathrm{~mm} / \mathrm{s}]$ | $z[\mathrm{~mm} / \mathrm{s}]$ | total $[\mathrm{mm} / \mathrm{s}]$ |
| :---: | :---: | :---: | :---: | :---: |
| KCSF | 51.88 | 36.83 | 116.4 | 205.1 |
| DIA+HPF | 27.45 | 34.91 | 48.97 | 111.3 |
| KCSF+DIA | 27.89 | 17.23 | 70.56 | 115.7 |
| Previous | 18.79 | 14.71 | 51.89 | 85.39 |
| KCP | 36.01 | 29.82 | 31.81 | 97.64 |
| Proposed | 12.10 | 11.06 | 25.15 | 48.32 |

is about twice as much as that of Proposed. This is because of combining with the accelerometer in the position estimator. Finally, compared Proposed with KCSF, DIA+HPF and KCSF+DIA, the root-mean-square error of Proposed is reduced about 50[\%]. Also, compared with Previous, it is reduced about $30[\%]$. Therefore, we conclude that Proposed is the best estimation in a comprehensive way.

## VI. CONCLUSION

This paper proposes a novel dead reckoning for biped robots which is available for high-rate feedback control. It is based on a dual-stage complementary filter consisting of the position estimator and the velocity estimator. The former estimator combines DIA with the trunk position calculated by KC and the latter does the integral of the acceleration with the estimated trunk position to make the assumptions of estimator consistent. We improve the accuracy of KC by using a point named MVP as the basis of KC, instead of the fixed support foot. The relative reliability between the signals varies with the variation of contact condition, so that the crossover frequency of those estimator is adaptively designed in accordance with the reaction force. The simulation result shows that the proposed dead reckoning reduces the root-mean-square error compared with other method.

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[^0]:    *This work was supported in part by Grant-in-Aid for Young Scientists(A) \#22680018, Japan Society for the Promotion of Science
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