

# Dead Reckoning of Biped Robots with Estimated Contact Points Based on the Minimum Velocity Criterion

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**Abstract**—A novel technique of dead reckoning for biped robots, which could be utilized for agile motion controls, is proposed. A complementary filter combines the estimations of the position of robot body from both the kinematic computation and the acceleration information, where the former is relied on in the lower frequency domain and the latter is done on in the higher domain. Even though the supporting foot of the robot happens to roll and rotate on the terrain, the estimation accuracy of the kinematics is improved by taking such movements into consideration. We suppose that the contact point moves with respect to the ground at the instantaneously minimum velocity, and thus name it the instantaneous minimum velocity point (IMVP), which is estimated by an optimization. IMVP can be computed for each foot, so that the weighted sum of them by the magnitude of reaction forces on each foot is adopted as a candidate of the contact point under an assumption that the contact condition is more steady when a larger reaction force is applied. Finally, it is merged with the twice-integrated acceleration through the complementary filter, where the crossover frequency is also determined by the reaction forces. Hence, it is robust against the change of contact conditions. Results of computer simulations will show that the proposed method reduces the estimation error comparing with the conventional methods.

## I. INTRODUCTION

An accurate position estimation with respect to the inertial frame is a crucial issue for mobile robots such as wheeled robots and legged robots. In order to measure the global position, external sensors such as cameras[1], laser range finders[2] and a combination of them[3] are commonly used. The sampling rate of those sensors is around tens of milliseconds, so that much faster but accurate position estimation is required for agile motion controls. Dead reckoning, in which a robot localizes itself only by internal sensors, is the most possible option.

The dead reckoning is a common technique in the field of wheeled robots[4]. Rotary encoders attached to wheels count the number of rotation and the travel distance is estimated. It will be more accurate when combined with other measurement devices such as cameras[5], GPS[6] and laser range finders[7]. On the other hand, the dead reckoning for legged robots has not been sufficiently discussed.

Legged robots move on the ground by exchanging the supporting leg alternatively. If the supporting foot ideally

keeps a steady contact to the terrain during a step, the relative movement of the body with respect to the ground can be computed from joint displacements, which are easily measured by encoders, through the forward kinematics[8][9]. In reality, however, the supporting foot happens to roll and rotate on the terrain. In such situations, the estimation accuracy is severely reduced. Some methods to combine it with external sensors as well as for wheeled robots have also been proposed[10], while they also include the problem of the low sampling rate. Another idea is to use an accelerometer mounted on the robot body[11], which is free from the contact condition of the supporting foot. It requires to integrate the sensor signal twice in order to obtain positional information, so that it necessarily suffers from the accumulation of errors mainly due to the drift of signals. In order to reduce that accumulation, the signal is combined with the above kinematics[12] by Kalman filter. However, Kalman filter has a difficulty to tune its parameter in general. Furthermore, this method assumes that the robot is always stably supported by three contact points, which is not available for the above situations.

This paper proposes a novel technique of dead reckoning for biped robots on the above issue. It is basically a complementary filter[14] combining the estimations from the kinematic computation and the acceleration information, where the former is relied on in the lower frequency domain and the latter is on in the higher domain. In order to improve the estimation accuracy of the kinematics in spite of the moving supporting foot, the movement, namely, rolling and rotation in particular, is taken into consideration. We suppose that the contact point moves with respect to the ground at the instantaneously minimum velocity, and thus name it the instantaneous minimum velocity point (IMVP), which is estimated by an optimization. IMVP can be computed for each foot, so that the weighted sum of them by the magnitude of reaction forces on each foot is adopted as a candidate of the contact point under an assumption that the contact condition is more steady when a larger reaction force is applied. Finally, it is merged with the twice-integrated acceleration through the complementary filter, where the crossover frequency is also determined by the reaction forces. Hence, it is robust against the change of contact conditions. Results of computer simulations will show that the proposed method reduces the estimation error comparing with the conventional methods.

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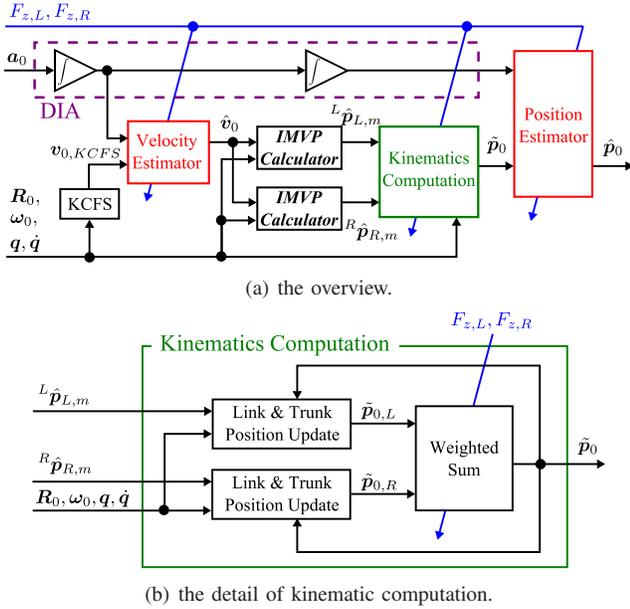


Fig. 1. The proposed dead reckoning

## II. DEAD RECKONING BASED ON INSTANTANEOUS MINIMUM VELOCITY POINT

### A. The previous dead reckoning

Legged robots move on the ground by exchanging the supporting leg alternatively. The forward kinematics between the supporting foot and the trunk is written as follows;

$$\mathbf{p}_S = \mathbf{p}_0 + \mathbf{R}_0^0 \mathbf{p}_S, \quad (1)$$

$$\mathbf{v}_S = \mathbf{v}_0 + [\boldsymbol{\omega}_0 \times] \mathbf{R}_0^0 \mathbf{p}_L + \mathbf{R}_0^0 \mathbf{v}_L, \quad (2)$$

$$\mathbf{R}_S = \mathbf{R}_0^0 \mathbf{R}_S, \quad (3)$$

$$\boldsymbol{\omega}_S = \boldsymbol{\omega}_0 + \mathbf{R}_0^0 \boldsymbol{\omega}_S, \quad (4)$$

where the subscripts  $S$  and  $0$  mean the support foot frame  $\Sigma_S$  and the body frame  $\Sigma_0$ , respectively.  $\mathbf{p}_*$ ,  $\mathbf{v}_*$ ,  $\mathbf{R}_*$  and  $\boldsymbol{\omega}_*$  are the position, velocity, attitude and angular velocity of  $\Sigma_*$  with respect to the inertial frame  $\Sigma$ , respectively.  ${}^0\mathbf{p}_*$ ,  ${}^0\mathbf{v}_*$ ,  ${}^0\mathbf{R}_*$  and  ${}^0\boldsymbol{\omega}_*$  represent the position, velocity, attitude and angular velocity of  $\Sigma_*$  with respect to  $\Sigma_0$ , respectively. If the supporting foot ideally keeps a steady contact to the terrain during a step and  $\mathbf{R}_0$  and  $\boldsymbol{\omega}_0$  are estimated by inertial sensors such as gyroscopes in advance, then  $\mathbf{p}_0$  and  $\mathbf{v}_0$  can be obtained by the kinematics computation from the motion of the support foot with respect to the trunk (KCFS), which is often called the dead reckoning of legged robots[8][9]. However, the supporting foot happens to roll and rotate on the terrain in real situations, so that the estimation accuracy is severely degraded. Another idea to obtain positional information is the double integral of acceleration (DIA) [11], which necessarily suffers from the accumulation of the low frequency noise.

In order to improve the accuracy, the nonlinear Kalman filter to combine inertial measurement unit with the leg kinematics was proposed[12]. Although Kalman filter is designed in the time domain, it has difficulty to tune its

parameters based on the statistical property. Furthermore, the robot is assumed to be always stably supported by three contact points, which is not available for the above situations.

### B. The proposed dead reckoning

For the above problem, we propose a dead reckoning for the biped robots which combines the kinematics computation with the acceleration. The proposed dead reckoning is basically a complementary filter[14]. It is designed in the frequency domain, so that it is easier to tune its parameters than Kalman filter when the frequency characteristics is roughly known. DIA is reliable in high frequency domain, so that we employ the high-pass filter (HPF) for that signal. The other signal is filtered by a low-pass filter (LPF) designed in a complementary way. In order to improve the accuracy of the kinematics computation based on the contact foot, IMVP with respect to the ground is used as the basis of kinematics, which is available even when the supporting foot rolls or rotates. IMVP is a point which has the minimum velocity with respect to the inertial frame. It is estimated based on the velocity information through an optimization.

The overview of the proposed method is shown in Fig.1. First, the trunk velocity is estimated by the velocity estimator which combines the velocity obtained by KCFS with the integral of the acceleration in a complementary manner. Next, IMVP is computed based on the estimated trunk velocity. Then, as shown in Fig.1(b), the weighted sum of the trunk position with respect to IMVP of each foot is calculated. Finally, the position estimator combines the kinematics computation with DIA in a complementary manner. The crossover frequency of the velocity estimator and the position estimator should be related with the reliability of the kinematics computation and DIA, so does the weight of the kinematics computation on that of solidness of contact of each foot. Both are evaluated in accordance with the ground reaction force. The detail is shown in the later section.

## III. IMVP CALCULATION BASED ON THE DIFFERENTIAL KINEMATICS

Hereafter, variables are represented in a discretized way with the sampling time  $\Delta T$ . For example, a variable  $*$  at the time  $(k-1)\Delta T$  is denoted as  $*[k-1]$ . Exceptionally, a variable at the time  $k\Delta T$  is written simply as  $*$  without  $[k]$  for the reader's convenience. Also,  $\mathbf{0} \in \mathbb{R}^3$  denotes the zero vector and  $\mathbf{1} \in \mathbb{R}^{3 \times 3}$  means the identity matrix.

Suppose that the left foot is the support foot, the position and velocity of IMVP with respect to  $\Sigma$  on the left foot frame  $\Sigma_L$  are expressed as follows;

$$\mathbf{p}_{L,m} = \mathbf{p}_L + \mathbf{R}_L^L \mathbf{p}_{L,m}, \quad (5)$$

$$\mathbf{v}_{L,m} = \mathbf{v}_L + [\boldsymbol{\omega}_L \times] \mathbf{R}_L^L \mathbf{p}_{L,m} + \mathbf{R}_L^L \mathbf{v}_{L,m}, \quad (6)$$

where  $\mathbf{p}_{L,m}$  and  $\mathbf{v}_{L,m}$  denote the position and velocity of IMVP with respect to  $\Sigma$  obtained by the information of  $\Sigma_L$ , respectively.  ${}^L\mathbf{p}_{L,m}$  and  ${}^L\mathbf{v}_{L,m}$  represent those values with respect to  $\Sigma_L$ , respectively. If  ${}^L\mathbf{p}_{L,m}$  is found under the

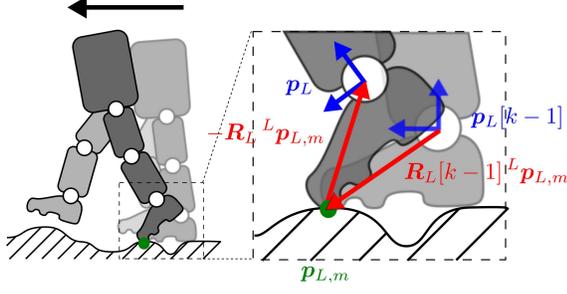


Fig. 2. The update of link position based on IMVP

condition that  $v_{L,m} \simeq 0$  in micro time, then the foot position can be updated based on Eqn. (5) as shown in Fig.2, namely,

$$\mathbf{p}_L = \mathbf{p}_{L,m} - \mathbf{R}_L^L \mathbf{p}_{L,m} \quad (7)$$

$$\simeq \mathbf{p}_L[k-1] + \mathbf{R}_L[k-1]^L \mathbf{p}_{L,m} - \mathbf{R}_L^L \mathbf{p}_{L,m}. \quad (8)$$

Therefore, the objective of this section is to compute  ${}^L \mathbf{p}_{L,m}$ . Assume that  ${}^L \mathbf{v}_{L,m} \simeq \mathbf{0}$ , Eqn.(6) is rewritten as

$$\mathbf{v}_{L,m} = \hat{\mathbf{v}}_L + [\hat{\boldsymbol{\omega}}_L \times] \hat{\mathbf{R}}_L^L \hat{\mathbf{p}}_{L,m}. \quad (9)$$

where  $\hat{\mathbf{v}}_0$  is the velocity estimate of trunk and  $\hat{\mathbf{v}}_L$  is the estimate of left foot's velocity obtained by Eqn.(2) as follows;

$$\hat{\mathbf{v}}_L = \hat{\mathbf{v}}_0 + [\hat{\boldsymbol{\omega}}_0 \times] \hat{\mathbf{R}}_0^0 \mathbf{p}_L + \hat{\mathbf{R}}_0^0 \mathbf{v}_L. \quad (10)$$

$\hat{\mathbf{R}}_0$  and  $\hat{\boldsymbol{\omega}}_0$  are able to be estimated by our previous work[13], so that the estimates  $\hat{\mathbf{R}}_L$  and  $\hat{\boldsymbol{\omega}}_L$  are also obtained by kinematics shown in Eqs.(3) and (4). A method to obtain the estimate  ${}^L \hat{\mathbf{p}}_{L,m}$  is to minimize  $\mathbf{v}_{L,m}$  represented in Eqn.(9), that is, to do the following evaluation function;

$$E_1 = \frac{1}{2} \|\hat{\mathbf{v}}_L + [\hat{\boldsymbol{\omega}}_L \times] \hat{\mathbf{R}}_L^L \hat{\mathbf{p}}_{L,m}\|^2. \quad (11)$$

The general solution, which satisfies the stationary condition  $\left(\frac{\partial E_1}{\partial {}^L \hat{\mathbf{p}}_{L,m}}\right)^T = \mathbf{0}$ , is computed as

$${}^L \hat{\mathbf{p}}_{L,m} = \frac{1}{{}^L \hat{\boldsymbol{\omega}}^T {}^L \hat{\boldsymbol{\omega}}} [{}^L \hat{\boldsymbol{\omega}} \times] \hat{\mathbf{R}}_L^T \hat{\mathbf{v}}_L + c \frac{{}^L \hat{\boldsymbol{\omega}}}{\|{}^L \hat{\boldsymbol{\omega}}\|}, \quad (12)$$

where  $c$  is a constant and

$${}^L \hat{\boldsymbol{\omega}} = \hat{\mathbf{R}}_L^T \hat{\boldsymbol{\omega}}_L. \quad (13)$$

However,  ${}^L \hat{\mathbf{p}}_{L,m}$  is not unique when  $\|{}^L \hat{\boldsymbol{\omega}}\| \rightarrow 0$ .

For this problem, the novel method to compute  ${}^L \hat{\mathbf{p}}_{L,m}$  is proposed. This computation takes the time variation of  ${}^L \hat{\mathbf{p}}_{L,m}$  into consideration, so that the evaluation function is redefined as

$$E = E_1 + \frac{1}{T_m^2} E_2, \quad (14)$$

where the second term in the right-hand side is the evaluation function which means the time variation of  ${}^L \hat{\mathbf{p}}_{L,m}$ , namely,

$$E_2 = \frac{1}{2} \|\mathbf{p}_{L,m} - {}^L \hat{\mathbf{p}}_{L,m}[k-1]\|^2. \quad (15)$$

$T_m$  is the positive time constant working as the weight. If  $T_m \rightarrow 0$ , then  $E \rightarrow E_2$  and  ${}^L \hat{\mathbf{p}}_{L,m}$  is nearly equal

to its initial value. On the other hand, if  $T_m \rightarrow \infty$ , then  $E \rightarrow E_1$  and  ${}^L \hat{\mathbf{p}}_{L,m}$  is nearly equal to the general solution represented by Eqn.(12) but the computation suffers from the above problem.

As well as  $E_1$ , the minimizer of  $E$  is computed under the stationary condition  $\left(\frac{\partial E}{\partial {}^L \hat{\mathbf{p}}_{L,m}}\right)^T = \mathbf{0}$ , so that

$${}^L \hat{\mathbf{p}}_{L,m} = \mathbf{C}_{1,L} \hat{\mathbf{R}}_L^T \hat{\mathbf{v}}_L + \mathbf{C}_{2,L} {}^L \hat{\mathbf{p}}_{L,m}[k-1], \quad (16)$$

where

$$\mathbf{C}_{1,L} = \frac{T_m^2}{\|{}^L \hat{\boldsymbol{\omega}}\|^2 T_m^2 + 1} [{}^L \hat{\boldsymbol{\omega}} \times], \quad (17)$$

$$\mathbf{C}_{2,L} = \frac{T_m^2}{\|{}^L \hat{\boldsymbol{\omega}}\|^2 T_m^2 + 1} \left( {}^L \hat{\boldsymbol{\omega}} {}^L \hat{\boldsymbol{\omega}}^T + \frac{1}{T_m^2} \mathbf{1} \right). \quad (18)$$

Note that IMVP position with respect to the right foot frame  $\Sigma_R$ , represented by  ${}^R \hat{\mathbf{p}}_{R,m}$ , is obtained similarly.

#### IV. THE IMPLEMENTATION OF THE PROPOSED DEAD RECKONING

##### A. The velocity estimator

In this section, the detail of the proposed dead reckoning for the implementation is described.

First, we compute the trunk velocity obtained by KCSF  $\mathbf{v}_{0,\text{KCFS}}$ . Suppose that the foot is the reference support foot for getting  $\mathbf{v}_{0,\text{KCFS}}$  when the vertical element of its reaction force  $F_{z,*}$  is greater than that of other,  $\mathbf{v}_{0,\text{KCFS}}$  is calculated from Eqn.(2) as follows;

$$\mathbf{v}_{0,\text{KCFS}} = \begin{cases} -[\hat{\boldsymbol{\omega}}_0 \times] \hat{\mathbf{R}}_0^0 \mathbf{p}_L - \hat{\mathbf{R}}_0^0 \mathbf{v}_L & (\text{if } F_{z,L} \geq F_{z,R}), \\ -[\hat{\boldsymbol{\omega}}_0 \times] \hat{\mathbf{R}}_0^0 \mathbf{p}_R - \hat{\mathbf{R}}_0^0 \mathbf{v}_R & (\text{otherwise}). \end{cases} \quad (19)$$

The integral of acceleration in high frequency domain is more reliable, so that it is filtered by HPF. On the other hand, the filter for  $\mathbf{v}_{0,\text{KCFS}}$  is designed in a complementary way. The estimate of velocity  $\hat{\mathbf{v}}_0$  is obtained by the following complementary filter;

$$\hat{\mathbf{v}}_0[k] = \mathbf{H}_{v1}(z) \mathbf{a}_{0,\text{mes}}[k] + \mathbf{H}_{v2}(z) \mathbf{v}_{0,\text{KCFS}}[k], \quad (20)$$

where,  $\mathbf{a}_0$  is the measurement of trunk acceleration with respect to the inertial frame.  $\mathbf{H}_{v1}(z)$ ,  $\mathbf{H}_{v2}(z)$  are the filters which are transformed from the following filters by the bilinear transformation.

$$\frac{1}{s} \cdot \mathbf{F}_{v1}(s) = \frac{1}{s} \cdot \frac{T_v s}{1 + T_v s} \mathbf{1}, \quad (21)$$

$$\mathbf{F}_{v2}(s) = \mathbf{1} - \mathbf{F}_{v1}(s) = \frac{1}{1 + T_v s} \mathbf{1}, \quad (22)$$

where  $T_v = 1/(2\pi f_v)$  and  $f_v$  is a crossover frequency of the velocity estimator.

##### B. The kinematics computation based on IMVP

In this subsection, we show the sequence of body position calculation by kinematics through IMVP. Suppose that the initial values of  $\Sigma_0$  (i.e.  $\mathbf{p}_0[0]$  and  $\mathbf{v}_0[0]$ ), that of each foot's IMVP  ${}^* \mathbf{p}_{*,m}[0]$  and that of the foot position  $\mathbf{p}_*[0]$  are

known. After IMVP calculation by Eqn.(16), the left foot position is updated by Eqn.(8), namely

$$\tilde{\boldsymbol{p}}_L = \hat{\boldsymbol{p}}_L[k-1] - \hat{\boldsymbol{R}}_L^L \hat{\boldsymbol{p}}_{L,m} + \hat{\boldsymbol{R}}_L[k-1]^L \hat{\boldsymbol{p}}_{L,m}, \quad (23)$$

where  $\tilde{\boldsymbol{p}}_L$  is the temporary estimate of left foot and  $\hat{\boldsymbol{p}}_L$  is the estimate of that. From Eqn.(1), the trunk position computed from the left foot  $\tilde{\boldsymbol{p}}_{0,L}$  is calculated as follows;

$$\tilde{\boldsymbol{p}}_{0,L} = \tilde{\boldsymbol{p}}_L - \hat{\boldsymbol{R}}_0^0 \boldsymbol{p}_L. \quad (24)$$

Likewise,  $\tilde{\boldsymbol{p}}_R$  and  $\tilde{\boldsymbol{p}}_{0,R}$  are also computed.

It is reasonable to obtain the temporary estimation of the trunk position  $\tilde{\boldsymbol{p}}_0$  from  $\tilde{\boldsymbol{p}}_{0,L}$  and  $\tilde{\boldsymbol{p}}_{0,R}$ , so that the following weighted average is employed.

$$\hat{\boldsymbol{p}}_0[k] = w_L \tilde{\boldsymbol{p}}_{0,L} + w_R \tilde{\boldsymbol{p}}_{0,R}. \quad (25)$$

When IMVP of one foot has less velocity than that of the other, we would like to make the weight for the trunk position calculated by its foot greater. Suppose that there are high possibility that such point exists in the support leg, the weight is calculated based on the reaction force  $F_{z,*}$ , that is,

$$w_* = \frac{\hat{F}_{z,*} + \epsilon_F}{\hat{F}_{z,L} + \hat{F}_{z,R} + 2\epsilon_F}, \quad (26)$$

$$\hat{F}_{z,*} = \begin{cases} Mg & (Mg < F_{z,*}) \\ F_{z,*} & (0 \leq F_{z,*} \leq Mg) \\ 0 & (F_{z,*} < 0) \end{cases}, \quad (* = L, R), \quad (27)$$

where  $\epsilon_F$  is a positive constant to make the denominator greater than zero. In this paper, we treat  $\epsilon_F$  as tuning parameter. Its effect is verified in the simulation.

There is the error between  $\hat{\boldsymbol{p}}_0$  and  $\tilde{\boldsymbol{p}}_{0,L}$ , so that it is accumulated with the passage of time. Thus, we correct it in updating  $\hat{\boldsymbol{p}}_L$  as follows;

$$\hat{\boldsymbol{p}}_L = \tilde{\boldsymbol{p}}_L + \Delta\hat{\boldsymbol{p}}_{0,L}, \quad (28)$$

where  $\Delta\hat{\boldsymbol{p}}_{0,L}$  is the correction of left foot position error calculated by

$$\Delta\hat{\boldsymbol{p}}_{0,L} = \hat{\boldsymbol{p}}_0 - \tilde{\boldsymbol{p}}_{0,L}. \quad (29)$$

Likewise,  $\hat{\boldsymbol{p}}_R$  is also corrected.

### C. The position estimator

The trunk position is estimated by combining  $\tilde{\boldsymbol{p}}_0$  with DIA as follows;

$$\hat{\boldsymbol{p}}_0 = \boldsymbol{H}_{p1}(z)\boldsymbol{a}_{0,mes} + \boldsymbol{H}_{p2}(z)\tilde{\boldsymbol{p}}_0, \quad (30)$$

where  $\boldsymbol{H}_{p1}(z)$  and  $\boldsymbol{H}_{p2}(z)$  is the filters that bilinear transformation transformed  $(1/s^2)\boldsymbol{F}_{p1}(s)$  and  $\boldsymbol{F}_{p2}(s)$  into, respectively.

$$\boldsymbol{F}_{p1}(s) = \frac{T_p^2 s^2}{1 + 2T_p s + T_p^2 s^2} \mathbf{1}, \quad (31)$$

$$\boldsymbol{F}_{p2}(s) = \mathbf{1} - \boldsymbol{F}_{p1}(s) = \frac{1 + 2T_p s}{1 + 2T_p s + T_p^2 s^2} \mathbf{1}, \quad (32)$$

where  $T_p = 1/(2\pi f_p)$  and  $f_p$  is a crossover frequency of the position estimator. DIA is reliable at high frequency, so that it is filtering by HPF  $\boldsymbol{F}_{p1}(s)$ . Also, LPF  $\boldsymbol{F}_{p2}(s)$  for  $\tilde{\boldsymbol{p}}_0$  is designed to satisfy the complementary condition.

### D. The determination of crossover frequency by the reaction force

The motion of the biped robot includes not only walking and standing, but also jumping. In the walking and standing, the kinematics computation is more reliable than DIA due to that one foot contacts on the ground at least. On the other hand, it is less reliable in the jumping. The reliability of each signal depends on the foot contact state, so that the crossover frequency of the filter can be adjusted in accordance with the reaction force if the contact points are assumed to be only on the foot. The reaction force is greater than zero when the foot contacts on the ground, while it is zero when there is no contact. Therefore, we determine the crossover frequency  $f$  based on the vertical element of total reaction force  $F_z$  as follows;

$$f(F_z) = \begin{cases} f_{\min} & (F_z \leq 0) \\ \hat{f}(F_z) & (0 \leq F_z \leq Mg) \\ f_{\max} & (Mg \leq F_z) \end{cases}, \quad (33)$$

where  $M$  is the mass of robot and  $g$  is the gravitational acceleration.  $f_{\min}$  and  $f_{\max}$  denote a maximum and a minimum crossover frequency.  $\hat{f}(F_z)$  is the monotone increasing function which satisfies the following conditions;

$$\hat{f}(0) = f_{\min}, \quad \hat{f}(Mg) = f_{\max}. \quad (34)$$

In this paper, we choose a linear function which satisfies the above conditions as  $\hat{f}(F_z)$ . The crossover frequency of above filters (i.e.  $f_p$  and  $f_v$ ) is designed by Eqn.(33).

## V. SIMULATION

### A. Setup

We used OpenHRP3[15] as the dynamic simulator. As shown in Fig.3, the used robot model has toe and heel joint in order to contact at not only sole but also toe and heel. We assumed that the reaction force of each foot is measured by the force sensor attached on the ankle. In the simulation, the robot walked forward with the toe and heel contact for 2 seconds and the sampling time is 2[ms]. The snapshots of the walking are shown in Fig.4. The joint torque for the motion was calculated by PD controller which is based on the referential angles and velocities of the joints. Those references were calculated in advance by the boundary condition relaxation proposed by Yamamoto et. al.[16]. The static and kinetic friction coefficient between the floor and robot model were set 1.0.

In the simulation, we compared the following methods;

- KCFS
- DIA with HPF (DIA+HPF)
- The complementary filter which combines KCFS with DIA (KCFS+DIA)
- The proposed dead reckoning (Proposed)

In order to examine the difference between the proposed dead reckoning and KCFS+DIA, the complementary filter of KCFS+DIA is the same as the position estimator shown in Eqs.(30),(31) and (32). The maximum and minimum crossover frequency of  $f_p$  were set 0.5[Hz] and 0.001[Hz].

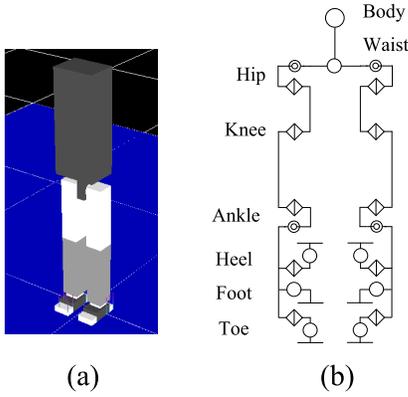


Fig. 3. The robot model for simulation (a)The exterior (b)The structure

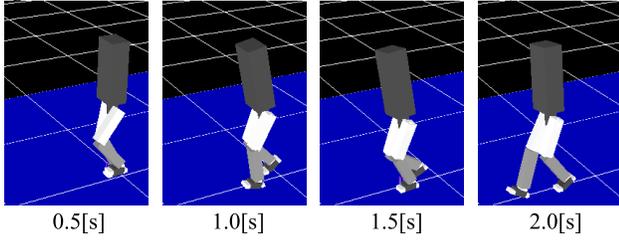


Fig. 4. The snapshots of simulation

Similarly, that of  $f_v$  are done 0.5[Hz] and 0.001[Hz]. In order to follow the robot movement, we employ the following filter as HPF of DIA+HPF.

$$F_{M2}(s) = \frac{(1/(0.002\pi)^2 s^2)}{1 + (1/(0.001\pi)s + (1/(0.002\pi)^2 s^2))}. \quad (35)$$

In order to examine the effect of the acceleration error, 10 offsets were only added to the true acceleration and we compared the above methods in the root-mean-square error (RMSE).

### B. Simulation result

First, we examined the effect of  $\epsilon_F$  by the position estimation at  $T_m = 10$ . The estimation result is shown in Fig.5. From the result, the error is least at  $\epsilon_F = 0.3$ [N], so that we use it hereafter.

Next, we examine the effect of  $T_m$ . The estimation result in some case of  $T_m$  is shown in Fig.6. The result shows that the error increases when  $T_m$  is small. On the other hand, when  $T_m$  becomes larger, the error seems to converge into a constant. Therefore, we decide to use  $T_m = 0.4$ [s] and  $\epsilon_F = 0.3$ [N].

The comparison results of position estimation and velocity estimation are shown in Table I and II, respectively. An example set of trajectories is plotted in Figs.7 and 8. The result of velocity estimation shows that DIA+HPF estimates more accurately than KCFS. This means that the velocity by KCFS is less reliable than the integration of acceleration in short term. On the other hand, in the position estimation, KCFS estimates more accurately than DIA+HPF. This is because the error is accumulated due to the integration. KCFS+DIA has the respective advantage to individual KCFS and DIA. Compared with them, the proposed method is more

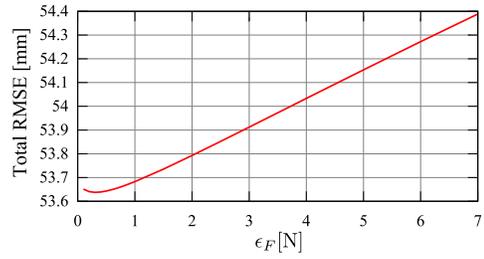


Fig. 5. RMSE of the position estimation varied with  $\epsilon_F$  at  $T_m = 10$

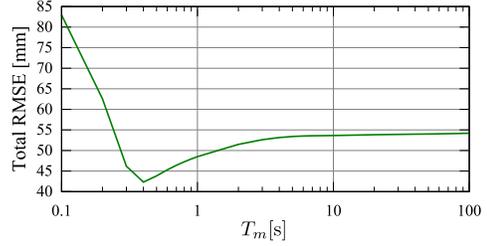


Fig. 6. RMSE of the position estimation varied with  $T_m$  at  $\epsilon_F = 0.3$

TABLE I  
ROOT-MEAN-SQUARE-ERROR OF THE ESTIMATED POSITION AT  
 $\epsilon_F = 0.3$ [N] AND  $T_m = 0.3$ [s] (UNIT:[MM])

| Method   | $x$   | $y$   | $z$   | total |
|----------|-------|-------|-------|-------|
| KCFS     | 26.77 | 4.202 | 30.73 | 61.71 |
| DIA+HPF  | 20.04 | 25.33 | 36.52 | 81.89 |
| KCFS+DIA | 23.70 | 4.967 | 34.00 | 62.66 |
| Proposed | 20.71 | 6.736 | 14.86 | 42.30 |

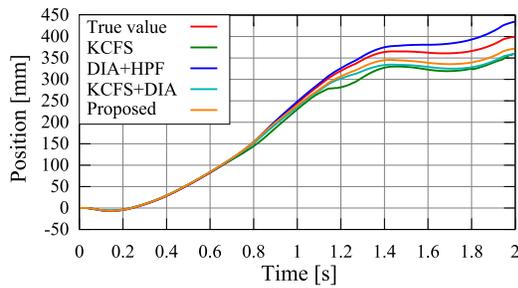
TABLE II  
ROOT-MEAN-SQUARE-ERROR OF THE ESTIMATED VELOCITY AT  
 $\epsilon_F = 0.3$ [N] AND  $T_m = 0.3$ [s] (UNIT:[MM])

| Method   | $x$   | $y$   | $z$   | total |
|----------|-------|-------|-------|-------|
| KCFS     | 75.67 | 33.80 | 101.4 | 210.8 |
| DIA+HPF  | 28.51 | 33.13 | 51.52 | 113.2 |
| KCFS+DIA | 33.02 | 16.92 | 58.59 | 108.5 |
| Proposed | 21.93 | 16.51 | 46.88 | 85.32 |

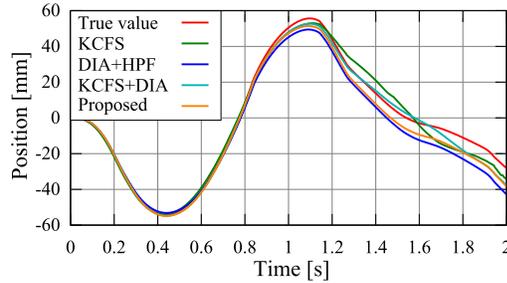
accurate than the other methods. In the position estimation, RMSE of proposed method is reduced about 30[%] compared with KCFS and KCFS+DIA. Especially, the error in  $z$ -direction is reduced more than about 50[%].

## VI. CONCLUSIONS

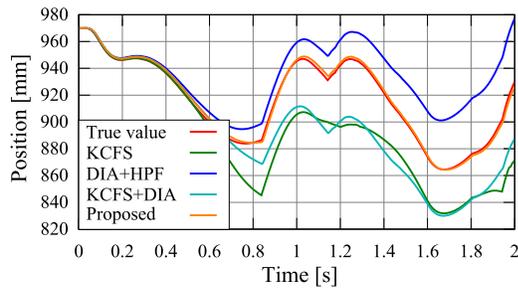
For the agile motion of the biped robots, this paper proposes a novel technique of the dead reckoning based on IMVP, which improves the kinematics computation. In this method, the trunk position obtained from each foot's IMVP is combined with DIA in a complementary manner. The crossover frequency of the filters and the weight of the kinematics computation are adaptively determined in accordance with the reaction force in order to reflect varying the reliability with the foot contact state. Through simulations, the proposed method reduces the position error about 30[%] compared with KCFS, DIA and the complementary filter which combines them.



(a) *x*-direction

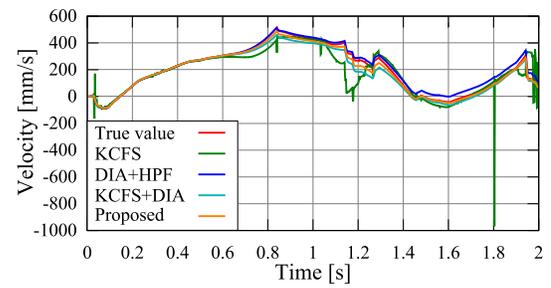


(b) *y*-direction

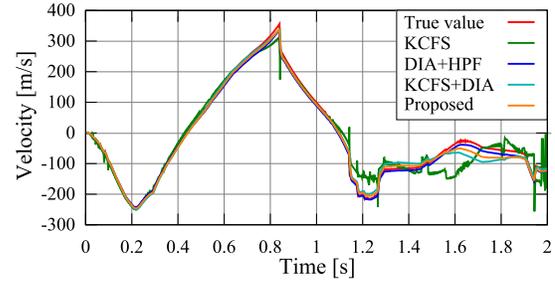


(c) *z*-direction

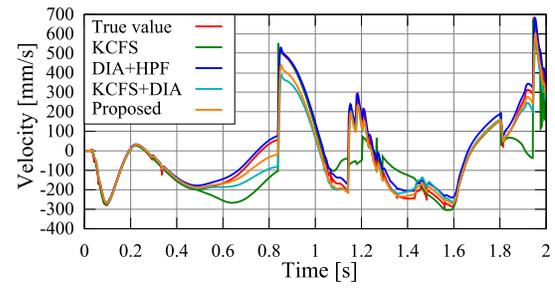
Fig. 7. A result of position estimation



(a) *x*-direction



(b) *y*-direction



(c) *z*-direction

Fig. 8. A result of velocity estimation

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